An Application of Realized Regression to the Futures Hedging Problem

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Abstract

We present a high-frequency based method for analyzing a one-period futures hedging problem. The realized hedge ratio is constructed by applying the realized regression with the R-squared coefficient as an ex-post performance measure. The asymptotic theory enables us to assess the parameter estimation risk of the hedge ratios. An empirical study is conducted on the S&P 500 index and their hedging performance is compared to the conventional hedge ratios. Moreover, impacts of the market microstructure effect on the realized hedge ratio and the effect of the parameter estimation risk on the corresponding hedging performance are discussed.

Keywords: Hedge ratio, hedging effectiveness, realized regression, parameter estimation risk

1. Introduction

Futures are standardized contracts traded in organized exchanges. As such, they are important hedging instruments for hedgers in eliminating the price risk of the spot position. To analyze the hedging problem, the mean-variance criterion is usually applied and the optimal hedge strategy is derived (Johnson, 1960; Ederington, 1979). If risk avoidance is the aim, the minimum-variance hedge ratio (MVHR) equals to the covariance between the spot and the futures returns divided by the variance of the futures return. It is a simplified version and thus is easier to be implemented at the empirical level.

There are models for estimating the MVHR, and the summarization with discussion is referred to Lien and Tse (2002). For example, the method of ordinary least squares (OLS) provides a static estimate. However, recognizing that returns typically exhibit time-varying volatilities and/or correlations, dynamic MVHR models are then developed by using some discrete-time multivariate volatility models, such as the vech-diagonal model of Bollerslev et al. (1988), the BEKK (named after Baba, Engle, Kraft, and Kroner) model of Engle and Kroner (1995), and the constant correlation generalized autoregressive conditional heteroscedasticity (CC-GARCH) model of Bollerslev (1990). Baillie and Myers (1991), Kroner and Sultan (1993), Brooks et al. (2002), and Lien et al. (2002) are examples for the applications. In comparing their performance, Ederington (1979) proposed the hedging effectiveness in evaluating the percentage reduction in the variance of the hedged portfolio relative to the unhedged spot position. Besides, alternative (unconditional) risk measures are also discussed, see, for example, Cotter and Hanly (2006).

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In this paper, the hedging problem is analyzed in a continuous-time framework. With some assumptions in the underlying price processes, we have the definition of integrated MVHR. The estimator, realized MVHR, is consistent with the integrated MVHR, and the estimate is achieved by applying the realized regression of Barndorff-Nielsen and Shephard (2004), using larger data sets of intraday returns. Meanwhile, the realized hedging effectiveness measure, or the R-squared coefficient in the realized regression, provides an ex-post estimate for the integrated hedging effectiveness. Furthermore, the asymptotic distribution provides insights into the precision of the realized MVHR. The realized method is then applied to the S&P 500 market spanning the period of January 2000 and December 2004 by assuming a weekly (Wednesday-based) hedging horizon. Moreover, its performance is compared to the conventional MVHR models.

Analyzing the hedging problem using the proposed method has the following advantages. First, the realized MVHR provides explicitly continuous evaluation in the integrated MVHR without assuming discrete-time multivariate volatility models or time variations in the MVHR dynamics. Meanwhile, the ex-post realizations enable us to study the properties within the MVHRs. Second, the conventional MVHRs are estimated by the parameter certainty equivalent (PCE) procedure. Substituting sample estimates of models’ parameters, the estimated MVHRs neglect the issue of parameter estimation risk (Lence and Hayes, 1994a, 1994b; Shi and Irwin, 2005). In contrast, the asymptotic confidence interval allows exploring the estimation risk at the empirical level. Third, the integrated volatility corresponds closely to the conditional volatility for discrete-time returns. As such, the realized hedging effectiveness provides an ex-post measure for conditional hedging effectiveness, as compared with the unconditional version of Ederington (1979).

The empirical findings are summarized as follows. First, 15-minute returns may be suitable for constructing realized weekly MVHRs for the S&P 500 index. The market microstructure effect leads the realized MVHR downward-biased when the sampling frequency chosen is high, such as 5-minute returns. Second, the realized weekly MVHR is generally superior to other conventional MVHRs, meanwhile, the GARCH MVHR do not perform better than the OLS estimate, based on realized hedging effectiveness measures. They are evaluated for the in-sample and out-of-sample period. Third, a regression analysis indicates that the realized hedging effectiveness is lower if the realized weekly MVHR becomes imprecise. The implication is that the parameter estimation risk is important when hedgers make hedging decisions. Therefore, the estimation risk should be incorporated into the MVHR models (Lence and Hayes, 1994a, 1994b; Shi and Irwin, 2005).

The paper is arranged as follows. Section 2 reviews the minimum-variance hedge strategy and the hedging effectiveness measure. Section 3 demonstrates the proposed method and Section 4 presents an empirical study. Finally, the conclusion is in Section 5.

2. Minimum-variance hedge strategy

Consider a one-period hedge problem. A hedger who simultaneously longs a spot portfolio and shorts a proportion of futures contracts at the beginning would like to reduce the price risk of the spot at the end of the period. Usually, nearby contracts are used due to liquidity concerns. Denote $p_t = \{p_t^f, p_t^s\}$ the 2×1 logarithmic price vector for the futures and the spot at day $t$. Then $r_t := p_t - p_{t-h}$ is the continuously compounded $h$-day return. Usually, $h = 1$ represents the daily return. If the hedger uses a hedge ratio $\beta_t$ for the hedging horizon, then we have the hedged portfolio return,

$$r_{t+1}^h := r_{t+1} - \beta_t r_{t+1}^f.$$ (1)
Note that this return is previously unknown and thus is a random variable at day $t$.

If the variance acts as the risk measure, then minimizing the variance of the hedged portfolio yields the MVHR estimator,

$$\beta_i = \frac{\text{cov}(r_{t+1}^f, r_t^s)}{\text{var}(r_t^f)}.$$ (2)

Approaches for estimating the MVHR is usually relied on the static (Naïve and OLS) or the dynamic (GARCH-type) models (see, for example, the discussion of Lien and Tse (2002)). The Naïve hedge ratio uses $\hat{\beta}_{\text{Naïve}} = 1$ for all the historical and forecasting period. Recognizing the comovement of the spot and the futures is not perfect, the OLS hedge ratio is obtained by running the linear regression using a set of $h$-day historical return data,

$$r_t^s = \alpha + \beta^{\text{OLS}} r_t^f + \epsilon_t,$$ (3)

and $\hat{\beta}^{\text{OLS}} = \text{cov}(r^f, r^s)/\text{var}(r^f)$ provides a static MVHR estimate for the historical period, and this becomes the MVHR forecast for all the future period.

A large body of studies, however, does not support the assumption of time invariant second moments in the asset return. For example, Bollerslev (1986) proposed the GARCH model and recognized that the returns typically exhibit time-varying volatility. The results are then extended to the multivariate volatility models, such as the vech-diagonal model of Bollerslev et al. (1988) or the BEKK model of Engle and Kroner (1995). These models are then broadly applied to the development of dynamic MVHR models (see, for example, Baillie and Myers (1991), Kroner and Sultan (1993), and Brooks et al. (2002)). In this study, we adapted the CC-GARCH model of Bollerslev (1990) to estimate the dynamic hedge ratios due to the benefit of convenience in the optimization process and the positive semi-definite in the correlation matrix. The specification of the CC-GARCH(1,1) model is,

$$r_{i,t} = \mu_i + \epsilon_{i,t} h_{i,t}^{1/2},$$

$$h_{i,t} = \alpha_{i,0} + \alpha_{i,1} h_{i,t-1} + \alpha_{i,2} \epsilon_{i,t-1}^2,$$ (4)

$$h_{f,t} = \rho h_{f,t-1} h_{s,t-1}^{1/2}.$$ (5)

$\hat{\beta}_i^{\text{GARCH}} = h_{f,t}/h_{f,t}$ provides MVHR estimates for the in-sample period. For the one-step-ahead forecasting, the CC-GARCH (1,1) gives $\hat{\beta}_t^{\text{GARCH}} = h_{f,t+1}/h_{f,t+1}$.

To compare the performance of MVHR models, Ederington (1979) proposed the hedging effectiveness ($HE$) measure,

$$HE := 1 - \frac{\text{var}(r_{t+1}^p)}{\text{var}(r_{t+1}^s)}.$$ (6)

This evaluates the percentage reduction in the variance of the hedged portfolio relative to the unhedged spot position. Besides, alternative risk measures are used, such as value-at-risk is one application.
3. The realized method

This section analyzes the hedging problem in a continuous-time scheme. Applying the concept of realized volatility, the realized MVHR is consistent with the integrated MVHR. Moreover, the asymptotic theory provides the limiting distribution for the integrated MVHR, and the R-squared coefficient in the realized regression gives the alternative for evaluating the MVHR models.

3.1 Realized variance and covariance

Suppose the logarithmic vector price, \( p_t \), follows a bivariate continuous-time stochastic volatility diffusion,

\[
dp_t = \mu_t dt + \Omega_t dW_t,
\]

where \( W_t \) is a standard 2-dimensional Brownian motion. The \( 2 \times 2 \) positive definite diffusion matrix, \( \Omega_t \), and the 2-dimensional instantaneous drift, \( \mu_t \), are strictly stationary and jointly independent of \( W_t \). Note that the diffusion matrix, \( \Omega_t \), consists of futures variance, \( \sigma^2_{f,f} \), spot variance, \( \sigma^2_{s,s} \), and covariance with futures and spot, \( \sigma_{f,s} \).

Andersen et al. (2001a) showed the limiting distribution of \( h \)-day continuously compounded returns. Conditional on the sample path realization of \( \mu_t \) and \( \Omega_t \),

\[
r_{t+h} | \sigma_{f,s}^t, \mu_{t+h}, \Omega_{t+h}^h \sim N \left( \int_0^h \mu_{t+h} d\tau, \int_0^h \Omega_{t+h} d\tau \right).
\]

The integrated diffusion matrix, \( \int_0^h \Omega_{t+h} d\tau \), provides a natural measure of the true latent \( h \)-day volatility. Moreover, by the theory of quadratic variation, Andersen et al. (2001a, 2001b, 2003) proposed the realized variance and covariance measure for the integrated latent volatility. As the sampling frequency of returns goes to infinity, or \( \Delta \to 0 \), we have

\[
\sum_{j=0}^{h/\Delta} r_{t+j\Delta}^\prime \Delta, r_{t+j\Delta} \Delta \to \int_0^h \Omega_{t+h} d\tau,
\]

almost surely for all \( t \).

3.2 Realized MVHR

With the underlying price process of Equation (7), we have the integrated MVHR,

\[
\beta^t := \int_0^h \frac{\sigma_{f,s}}{\sigma_{f,s}^2} d\tau
\]

This result is also found in Harris et al. (2007), and a similar study on the integrated beta of the one-factor capital asset pricing model (CAPM) is referred to Andersen et al. (2006). To estimate the integrated MVHR, the realized MVHR provide an \textit{ex-post} method,
\[
\hat{\beta}_t^{\text{Real}} = \sum_{j=1}^{b/\Delta} \left( r_{t+j, \Delta, \Delta}^f r_{t+j, \Delta, \Delta}^s \right) \rightarrow \beta_t,
\]

(11)

as \( \Delta \rightarrow 0 \), or the realized regression gives the same estimate (Barndorff-Nielsen and Shephard, 2004),

\[
r_{t+j, \Delta, \Delta}^s = \beta_t^{\text{Real}} r_{t+j, \Delta, \Delta}^f + \epsilon_{t+j, \Delta, \Delta}, \quad j = 1, \ldots, h/\Delta.
\]

(12)

The asymptotic property for the realized regression is also discussed in Barndorff-Nielsen and Shephard (2004). If the realized MVHR has this simple form, then the asymptotic distribution of the integrated MVHR is,

\[
\frac{\hat{\beta}_t^{\text{Real}} - \beta_t}{\sqrt{\left( \sum_{j=1}^{b/\Delta} \left( r_{t+j, \Delta, \Delta}^f \right)^2 \right)^{-2} \hat{\epsilon}_t}} \sim N \left( 0, 1 \right),
\]

(13)

Where

\[
\hat{\epsilon}_t := \sum_{j=1}^{b/\Delta} a_j^2 - \sum_{j=1}^{b/\Delta-1} a_j a_{j+1},
\]

(14)

and

\[
a_j := r_{t+j, \Delta, \Delta}^s - \hat{\beta}_t^{\text{Real}} \left( r_{t+j, \Delta, \Delta}^f \right)^2 - \hat{\beta}_t^{\text{Real}} \left( r_{t+j, \Delta, \Delta}^f \right)^2.
\]

(15)

With the result of Equation (13), we then have the \( \alpha \)% asymptotic confidence interval for the integrated MVHR,

\[
\beta_t^{\text{Real}} \in \hat{\beta}_t^{\text{Real}} \pm z_{\alpha/2} \frac{\sqrt{\left( \sum_{j=1}^{b/\Delta} \left( r_{t+j, \Delta, \Delta}^f \right)^2 \right)^{-2} \hat{\epsilon}_t}}{\sqrt{\left( \sum_{j=1}^{b/\Delta} \left( r_{t+j, \Delta, \Delta}^f \right)^2 \right)^{-2} \hat{\epsilon}_t}},
\]

(16)

where \( z_{\alpha/2} \) is the critical value of a standard normal distribution.

3.3. Realized hedging effectiveness

Using the concept of realized volatility, the realized hedging effectiveness (RHE) provides an alternative \textit{ex-post} method for comparing various MVHR models,

\[
RHE_t := 1 - \frac{\sum_{j=1}^{b/\Delta} \left( r_{t+j, \Delta, \Delta}^p \right)^2}{\sum_{j=1}^{b/\Delta} \left( r_{t+j, \Delta, \Delta}^s \right)^2} \rightarrow HE_t := 1 - \frac{\int_0^h \sigma_{d, \Delta, \Delta}^2 \, dt}{\int_0^h \sigma_{s, \Delta, \Delta}^2 \, dt},
\]

(17)
Note that the $RHE$ is equivalent to R-squared coefficient if the MVHR estimate is applied to the realized regression of Equation (12).

4. Empirical application

An empirical study on the mid-week (Wednesday-base) hedging analysis is presented in this section. The underlying assets contain the S&P 500 index and the S&P 500 futures traded on the Chicago Mercantile Exchange (CME). Data sets contain weekly and high frequency intraday prices spanning the period of January 2000 and December 2004. The continuous price series for the futures are created when the 2nd month future volume exceeds the 1st future month volume. The data are provided by the Tick Data and Datastream. The analysis is divided into three parts, including the proper sampling frequency for the realized weekly MVHR, the evaluation of MVHR models, and the impact of the parameter estimation risk on the corresponding hedging performance.

4.1 Sampling frequencies and realized MVHRs

The issue of sampling frequency chosen should be important for calculating the realized weekly MVHR. For example, due to the market microstructure noises, Barndorff-Nielsen and Shephard (2002) showed that the realized daily volatility is biased when the sampling frequency is high. Thus, we study the properties of realized weekly MVHR against various sampling frequencies.

Table 1 summarizes the statistics of realized weekly MVHRs against five sampling frequencies, including 5-, 10-, 15-, 30-, and 60-minute. Applying these data sets to Equation (11), we have 261 realized weekly MVHRs for each sampling method. The statistics in Table 1 conclude that all the MVHRs exhibit time-varying, left-skewed and leptokurtic. Moreover, the means of the realized MVHRs are about 0.90 except the result obtained from the 5-minute data. For example, a two-tail t statistic (-22.17) indicates that the mean of the 5-minute and the 15-minute realized MVHRs are different at the 1% significance level. The preliminary analysis shows that the realized weekly MVHRs may be biased estimates for the integrated weekly MVHRs if 5-minute returns are used.

Table 1. Summary statistics of the realized weekly MVHRs against sampling frequencies

<table>
<thead>
<tr>
<th>Δ (Minute)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>0.5552</td>
<td>0.6779</td>
<td>0.7554</td>
<td>0.4107</td>
<td>0.3273</td>
</tr>
<tr>
<td>25th</td>
<td>0.7972</td>
<td>0.8749</td>
<td>0.8925</td>
<td>0.8678</td>
<td>0.8717</td>
</tr>
<tr>
<td>50th</td>
<td>0.8332</td>
<td>0.9014</td>
<td>0.9219</td>
<td>0.9052</td>
<td>0.9286</td>
</tr>
<tr>
<td>75th</td>
<td>0.8565</td>
<td>0.9206</td>
<td>0.9413</td>
<td>0.9419</td>
<td>0.9816</td>
</tr>
<tr>
<td>Max</td>
<td>0.9319</td>
<td>0.9779</td>
<td>1.1448</td>
<td>1.0374</td>
<td>1.1261</td>
</tr>
<tr>
<td>Mean</td>
<td>0.8229</td>
<td>0.8959</td>
<td>0.9176</td>
<td>0.8989</td>
<td>0.9218</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0535</td>
<td>0.0394</td>
<td>0.0409</td>
<td>0.0659</td>
<td>0.0920</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.3426</td>
<td>-1.1587</td>
<td>-0.0592</td>
<td>-2.0097</td>
<td>-1.5209</td>
</tr>
<tr>
<td>Obs.</td>
<td>261</td>
<td>261</td>
<td>261</td>
<td>261</td>
<td>261</td>
</tr>
</tbody>
</table>

A proper sampling frequency for the realized weekly MVHR may be addressed by plotting the MVHR signature plot. This provides a visual trade-off between the accuracy and the precision of the realized weekly MVHR, and the spirit is similar to the volatility signature plot of Andersen et al. (2000). In this study, the accuracy is measured by the mean of the realized weekly MVHRs, and the precision is measured by the mean of the 95% asymptotic
confidence intervals. Figure 1 shows the plot. It can be observed that the 95% asymptotic confidence interval narrows when the sampling frequency increases. That is, the realized MVHR becomes precise if a larger number of observations are used. The smallest two confidence intervals, 0.1147 and 0.1184, occur in the 5-minute and the 15-minute, respectively, and the highest, 0.2401, occurs in the 60-minute. In balancing the accuracy and the precision of the realized weekly MVHR, it seems that the 15-minute MVHR is more reliable.

![Figure 1](image)

Figure 1. Average values of the realized weekly MVHRs and 95% confidence intervals drawn against sampling frequencies.

The sampling method chosen also influences the dynamic properties of the realized weekly MVHR. For example, Figure 2 plots the 5-minute and the 15-minute realized weekly MVHRs in panels (a) and (b), respectively. The solid line represents the realized weekly MVHR, and the dotted lines represent the corresponding 95% asymptotic confidence intervals. Both MVHRs are fluctuated as the market environment changes; however, their dynamics are quite different. The 15-minute realized MVHRs reveal mean reverting from years 2000 to 2004, but the 5-minute realized MVHRs reveal an upward movement before year 2002. A dummy regression analysis,

\[ \beta_i^2 = 0.8513 - 0.0471D + u_i^5, \quad R^2 = 0.19, \]

\[ (177.05) \quad (-7.72), \]

shows a structural change in the unconditional mean of the 5-minute realized weekly MVHRs, where the dummy variable \( D \) equals one if the MVHR is before year 2002; otherwise, it equals zero. Figure 3 shows the sample autocorrelation functions for the 5-minute and the 15-minute MVHR. The result indicates that the 15-minute MVHR has weaker time-dependence, compared with the 5-minute MVHR.
Figure 2. Time series plots of realized weekly MVHRs with 95% asymptotic confidence intervals.

Figure 3. Sample autocorrelations of realized weekly MVHRs with 95% Bartlett’s confidence interval: \[\pm 1.96/\sqrt{261}\].

4.2 Performance evaluation results

The benefit of using larger intraday data sets in estimating weekly MVHRs is compared with traditional approaches, including the Naïve, static OLS, and dynamic CC-GARCH(1,1) MVHRs. They are evaluated for both in-sample and out-of-sample, based on the proposed RHE of Equation (17). The sample period, January 2000 to December 2003, is used for the in-sample estimation (209 weeks), which leaves one year for the out-of-sample comparisons (52 weeks). The comparison results are shown in Table 2.
Table 2. Unconditional sample means of the realized weekly hedging effectiveness.

<table>
<thead>
<tr>
<th>Δ (Minute)</th>
<th>Naïve</th>
<th>OLS</th>
<th>GARCH</th>
<th>Realized</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In-sample (209 obs.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.7708</td>
<td>0.7830</td>
<td>0.7828</td>
<td>0.7984</td>
</tr>
<tr>
<td>10</td>
<td>0.8828</td>
<td>0.8885</td>
<td>0.8877</td>
<td>0.8924</td>
</tr>
<tr>
<td>15</td>
<td>0.9169</td>
<td>0.9210</td>
<td>0.9198</td>
<td>0.9245</td>
</tr>
<tr>
<td>30</td>
<td>0.8857</td>
<td>0.8918</td>
<td>0.8911</td>
<td>0.8982</td>
</tr>
<tr>
<td>60</td>
<td>0.9028</td>
<td>0.9074</td>
<td>0.9070</td>
<td>0.9111</td>
</tr>
<tr>
<td><strong>Out-of-sample (52 obs.)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.7881</td>
<td>0.7995</td>
<td>0.7967</td>
<td>0.8135</td>
</tr>
<tr>
<td>10</td>
<td>0.8920</td>
<td>0.8974</td>
<td>0.8954</td>
<td>0.9008</td>
</tr>
<tr>
<td>15</td>
<td>0.9227</td>
<td>0.9267</td>
<td>0.9245</td>
<td>0.9297</td>
</tr>
<tr>
<td>30</td>
<td>0.8887</td>
<td>0.8947</td>
<td>0.8922</td>
<td>0.9007</td>
</tr>
<tr>
<td>60</td>
<td>0.9058</td>
<td>0.9103</td>
<td>0.9083</td>
<td>0.9137</td>
</tr>
</tbody>
</table>

The in-sample MVHRs for the OLS and the CC-GARCH are estimated by Equation (3) and Equations (4)-(5), respectively, using the 209 weekly returns. The OLS gives static estimates, 0.96, while the CC-GARCH gives time-varying estimates ranging from 0.86 to 1.06. Meanwhile, the 15-minute realized weekly MVHRs are used for the comparisons, and the Naïve one-to-one is regarded as a benchmark. With 15-minute ex-post returns, we have the corresponding 209 RHEs. Besides, we also apply the 5-, 10-, 30-, and 60-minute returns to Equation (17) to achieve a robust result, and the unconditional sample means of the RHEs for each MVHR model are reported in Table 2. The result concludes that the realized MVHR yields the best performance for the in-sample. Besides, the superior OLS than the CC-GARCH model agrees with previous researches, see, for example, Lien et al. (2002).

For the out-of-sample evaluations, the one-step-ahead forecasts for the Naïve, OLS, and CC-GARCH MVHR are estimated using a rollover method. We keep the fixed estimation sample size 209 (observation for time T+1 is incorporated and the observation for 1 is deleted) and three series of MVHR forecasts are then obtained. For the realized MVHR, the lagged one period realized MVHR is applied as the proxy for the one-step-ahead forecast. The average RHEs for each MVHR model are compared. The out-of-sample result in Table 2 concludes that the realized MVHR yields the best performance.

4.3 Impact of parameter estimation risk

Whether the issue of parameter estimation risk affects the hedging performance is of much interest. This issue is then analyzed by a time-series regression. Note that the control variable, realized weekly MVHR, is included in the regression due to the definition of R-squared in the realized regression, and the \( ASD \) represents the asymptotic standard error in Equation (16). Using the 261 weeks of the ex-post 15-minute realized measures, we have,

\[
RHE_{i}^{15} = 0.7536 + 0.2477 \beta_{i}^{15} - 1.6848 ASD_{i}^{15} + u_{i}^{15} \quad R^2 = 0.50
\]

\[
(18.77) \quad (5.76) \quad (-14.06)
\]

and the statistical relation between the realized MVHR and the \( ASD \) is also reported,
In addition, the time series plots of \( RHEs \) (constructed from the 5-, and the 15-minute) are shown in Figure 4. The empirical analysis concludes that the hedging performance is lower upon increasing the estimation risk of MVHR. Similar results are concluded for the time series regression using the x-minute realized measures. The implication of these results is that the issue of the parameter estimation risk for estimating the MVHR is important so that the risk should be incorporated into the MVHR models (Lence and Hayes, 1994a, 1994b; Shi and Irwin, 2005).

Figure 4. Time series plots for the realized weekly hedging effectiveness measures.

5. Conclusion

A high-frequency based method for the one-period futures hedging problem has been presented in this article. Applying the realized regression method of Barndorff-Nielsen and Shephard (2004), the realized MVHR provides explicitly continuous evaluation for the integrated MVHR, and the realized hedging effectiveness is \textit{ex-post} realization for the hedging performance. The asymptotic confidence interval further gives insights to the issue of parameter estimation risk. The realized method is applied to the S&P 500 index and an empirical study is then conducted.

The empirical study discusses the optimal sampling frequency for computing the realized MVHR. The evidence shows that the 15-minute returns are suitable for calculating the realized weekly MVHRs. The market microstructure effect will make them to be downward-biased. For comparing the performance of realized MVHRs with conventional MVHR estimates, it is found that the benefit of using larger intraday data sets is substantial. The realized MVHRs generally yield higher realized hedging effectiveness than the conventional MVHR estimates, for both the in-sample (209 weeks) and the out-of-sample (52 weeks). This result is robust to the choice of sampling frequencies in computing the realized hedging effectiveness. Finally, the impact of the parameter estimation risk on the hedging performance is also discussed. The realized weekly hedging effectiveness decreases when the realized weekly MVHR is imprecise. The implication is that the parameter estimation risk is important for estimating the MVHR.
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