The Information Transmission between Two Substitutes of Index Futures: The Case of TAIEX and Mini-TAIEX Stock Index Futures

Ching-Chung Lin*, Hsinan Hsu** and Chwan-Yi Chiang***

Abstract

Past studies propose many possible explanations to the lead-lag relationship. However, it is not easy to clarify the contributions of contract differences, which simultaneously exist among those highly related financial securities, to the lead-lag phenomenon. The TAIEX and the Mini-TAIEX stock index futures are identical except for their contract sizes and the trading volume of the TAIEX index futures is much higher than that of the Mini-TAIEX. By employing the vector error correction model (VECM), Gonzalo-Granger information share, and generalized impulse response functions and variance decompositions, this paper finds that there exists a strong information transmission from the TAIEX index futures to the Mini-TAIEX index futures. This result suggests that, due to higher liquidity, the TAIEX index futures play a significantly important role in price discovery.

Keywords: Generalized impulse response functions; Information transmission; Information share; Index futures

1. Introduction

Because of the rapid development of financial engineering and the competition between financial markets, there are various kinds of financial securities and markets to satisfy investors’ various investing needs. If the markets were perfect, the prices of those highly related securities should simultaneously reflect new information. That is, there should be no lead-lag relationship in price movements from one market to another and no single security or market dominates the process of price discovery. However, because of market frictions, such as transaction costs and short-selling restrictions, trading mechanism, home market of financial asset, market

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liquidity, and other possible factors, significant lead-lag relationships between financial assets can be observed.

A number of empirical studies have examined the linkages and interactions between returns of stock and futures markets. Some notable examples include studies by Kawaller et al. [11], Ng [16], Stoll and Whaley [24], Chan [2], and Koutmos and Tucker [13]. Not only the lead-lag relationship in terms of returns, many studies have tested for a lead-lag in the volatility of returns between financial assets. Examples include Cheung and Ng [5], Chan et al. [3], Koutmos and Tucker [13], Tse [26], and among others. The relationship is of particular interest because of the theoretical links between information and volatility suggested by Ross [20]. In addition, the price discovery and information transmission of international markets have been intensively investigated, for examples, Booth et al. [1] for DAX index futures, Fleming et al. [7] for S&P index futures and options, Shyy and Lee [21] for bund futures in LIFFE and DTB markets, and Tse [25] for Euromark futures in LIFFE and SGX markets.

Past limited studies concerning Taiwan stock index futures focus on the topic of interaction between the Singapore Exchange (SGX) and the Taiwan Futures Exchange (TAIFEX). The SGX provides Taiwan index futures contract on the Morgan Stanley Capital International Taiwan Stock Index (MSCI-TW), while the TAIFEX index futures is on the Taiwan Stock Exchange (TSE) Capitalization Weighted Stock Index (TAIEX). Chou and Lee [6] find that, as the trading cost drops in Taiwan, the strength of information transmissions from the TAIFEX to the SGX increases significantly and that the information transmission is bilateral. Roope and Zurbrueg [19] demonstrate that both futures markets play a role in price discovery, but Singapore prices will tend to reflect new information first. Chang and Wang [4], however, report that no single market that dominated the production of market information.

Most of empirical studies investigate the relationship between or among highly related financial securities, for examples, spot, futures, and options markets, or securities listed in different exchanges. Although these markets are related, there still simultaneously exist many differences between them, including trading mechanism, contract specifications, trading costs, market liquidity, and so on. Researchers propose many possible explanations to the lead-lag phenomenon (for examples, see Chan [2] and Fleming et al. [7]), but it is not easy to clarify the contributions of these possible factors to the lead-lag relationship. However, this question can be examined using two
Taiwan stock index futures contracts.

After successfully introducing the TAIEX stock index futures, the TAIFEX announced to attract more individual investors, who contribute 84% of the TSE trading volume in 2001, by offering the Mini-TAIEX stock index futures. Basically, these index futures are identical securities. They have the same underlying index, trading mechanism, and many other trading conditions. The major difference between them is the contract size. The contract size of the Mini-TAIEX futures is only one-fourth of that of the TAIEX futures and, hence, the margin requirements and position limits are different. Because buying one contract of the TAIEX index futures equals buying four Mini-TAIEX futures, from the viewpoint of trading mechanism and contract specifications, these two index futures are substitutes for each other. However, because of longer history, the trading volume of the TAIEX futures is higher than that of the Mini-TAIEX futures.

While these two index futures are identical contracts, is it possible that one of them lead another one in revealing new information? Since these two TAIEX index futures of different contract sizes share the same trading mechanism and trading costs, if there does exist any lead-lag relationship between those two index futures, the hypotheses regarding trading mechanism and trading costs would not be able to provide a satisfactory explanation. However, although the announced targeted investors of the Mini-TAIEX index futures are the major forces of the TAIEX spot market, because of longer history and higher liquidity and, hence, possibly attracting better-informed investors, the prices of the TAIEX index futures might transmit more information than those of the Mini-TAIEX index futures does. Therefore, this empirical study is to investigate the intraday price discovery and information transmissions between these two index futures by utilizing the vector error correction model (ECM), information share developed by Gonzalo and Granger [8], and generalized impulse response function (GI) proposed by Koop et al. [12] and Pesaran and Shin [18].

The rest of this paper is organized as follows. The next section explains the specifications of the Mini-TAIEX and the TAIEX stock index futures. The section following is data description and preliminary statistics. The empirical methodology and results are presented in Sections 4 and 5, respectively. The final section concludes.

2. TAIEX Index Futures Contracts

The history of Taiwan futures market is short. Not until July 21, 1998
did the TAIFEX introduce its first own futures contract on the TAIEX, while Singapore began to offer an MSCI Taiwan futures contract in January 1997. Although the TAIEX index futures faced the competition from the SGX-DX MSCI Taiwan index futures, its daily volume steadily grew to an average 23,000 contracts in March 2002. To further satisfy the hedging demands of individual investors in the TSE and to make them be able to participate the futures market with less capital, the TAIFEX began offering the Mini-TAIEX index futures starting April 9, 2001. The average daily trading volume in March 2002 was around 3,600.

To understand why the TAIFEX launched the Mini-TAIEX futures trying to attract more investors into futures market, it is helpful to highlight the investor structure of the TSE. Table 1 shows the comparison of Taiwan and three major stock markets. At the end of 2001, the number of companies listed on the TSE was 584 and the trading value was $546.1 billions. Compared with the numbers of listed companies and trading values of New York (2,400, $8,945.2), London (2,891, $3,399.4) and Tokyo (2,103, $1,660.5), Taiwan’s figures (584, $546.1) are relatively low. However, Taiwan’s turnover of trading value (206.95%) is much higher than those of New York (87.62%), London (76.1%) and Tokyo (56.52%). The reason of Taiwan’s incredibly high turnover is the unique investor structure, which, on the TSE of 2001, was significantly dominated by domestic individual investors (84%), compared to domestic institutional investors (10%), and foreign institutional investors (6%). Because the individual investors, who contribute around 84% of trading value of the TSE, are the major force in the TSE, no wander why individual investors become the targeted market of the TAIFEX, and why the TAIFEX provides a smaller futures contract to attract those investors into the index futures market.

Table 1 Selected Statistics of Four Stock Markets

<table>
<thead>
<tr>
<th></th>
<th>Taiwan</th>
<th>New York</th>
<th>London</th>
<th>Tokyo</th>
</tr>
</thead>
<tbody>
<tr>
<td># of listed Co., December 31, 2001</td>
<td>584</td>
<td>2,400</td>
<td>2,891</td>
<td>2,103</td>
</tr>
<tr>
<td>Trading Value (USD billions), 2001</td>
<td>$546.1</td>
<td>$10,489.3</td>
<td>$4,550.5</td>
<td>$1660.5</td>
</tr>
<tr>
<td>Turnover of Trading Value, 2001</td>
<td>206.95%</td>
<td>87.62%</td>
<td>76.1%</td>
<td>56.52%</td>
</tr>
</tbody>
</table>

Source: Taiwan Securities Exchange Corporation
Table 2 Comparison of the TAIEX and the Mini-TAIEX Stock Index Futures

<table>
<thead>
<tr>
<th></th>
<th>TAIEX futures</th>
<th>Mini-TAIEX futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underlying Index</td>
<td>Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX)</td>
<td></td>
</tr>
<tr>
<td>Trading Hours</td>
<td>08:45AM-1:45PM Taiwan time Monday through Friday</td>
<td></td>
</tr>
<tr>
<td>Delivery Months</td>
<td>Spot month, the next calendar month, and the next three quarter months</td>
<td></td>
</tr>
<tr>
<td>Last Trading Day</td>
<td>The third Wednesday of the delivery month of each contract</td>
<td></td>
</tr>
<tr>
<td>Minimum Price Fluctuation</td>
<td>One index point</td>
<td></td>
</tr>
<tr>
<td>Daily Price Limit</td>
<td>±7% of previous day's settlement price</td>
<td></td>
</tr>
<tr>
<td>Contract Size</td>
<td>NT$200 × Index</td>
<td>NT$50 × Index</td>
</tr>
<tr>
<td>Initial Margin*</td>
<td>NT$120,000</td>
<td>NT$30,000</td>
</tr>
<tr>
<td>Maintenance Margin*</td>
<td>NT$92,000</td>
<td>NT$23,000</td>
</tr>
<tr>
<td>Position Limit (contracts)</td>
<td>1. Individuals: 300</td>
<td>1. Individuals: 600</td>
</tr>
<tr>
<td></td>
<td>2. Institutional investors: 1,000</td>
<td>2. Institutional investors: 2,000</td>
</tr>
</tbody>
</table>

* Effective on May 20, 2002
Source: Taiwan Futures Exchange

Table 2 compares the specifications of the TAIEX and the Mini-TAIEX futures contracts. These two stock index futures almost are identical securities. They have the same underlying index asset, contract months, delivery months, daily price limits, and other trading conditions. The major difference between them is the contract size. The contract size of the Mini-TAIEX index futures is only NT$50 times the TAIEX spot index, while that of the TAIEX index futures is NT$200 times the TAIEX spot index. The TAIEX index spot was around 6,000 points in May 2002 and, therefore, the contract sizes of the TAIEX and the Mini-TAIEX stock index futures were around NT$1,200,000 and NT$300,000, respectively. The contract size of Mini-TAIEX futures is only one-fourth of that of TAIEX index futures. Hence, their margin requirements and position limits are different.

3. Data Description and Preliminary Statistics

This article uses 1-minute intraday prices of the TAIEX and the Mini-TAIEX stock index futures data. To define the futures prices, instead of using the nearby contract, we use the most active one, which has the highest trading volume among five available contracts of different delivery months.
To obtain the 1-minute intraday prices of index futures, we use the price of the last transaction of each 1-minute interval to serve as the index futures price of this interval. If there is no transaction in a 1-minute period, the price of the previous interval will be used.

The sample period is February 20, 2002 to March 29, 2002 and the sample size is 7965. There is no any 1-minute interval in which no transaction happens in the price series of the TAIEX index futures. The number of no-transaction period of the Mini-TAIEX index futures is 69 and the ratio of no-transaction is only 0.87% (=69÷7965). The average span of actual trading time to 1-minute time point for the TAIEX and the Mini-TAIEX index futures are 5.6 and 10.5 seconds, respectively. Based on the figures of no-transaction ratio and their average trading time difference, the problems of infrequent and nonsynchronous trading will not be taken into account in our study.

Panel A in Table 3 shows the average daily transaction statistics of the TAIEX and the Mini-TAIEX stock index futures. Since the history of the Mini-TAIEX stock index futures is short, its transaction is not as active as that of the TAIEX stock index futures. The average daily trading volume of the Mini-TAIEX stock index futures is only 16% (=3,620÷22,981) of that of the TAIEX index futures and reaches 3,620 contracts. Compared with the average volume per transaction of the TAIEX index futures of 13.38, that of the Mini-TAIEX index futures of 2.81 is significantly lower at the significant level of 1%. Moreover, because the contract size of the Mini-TAIEX index futures is only one-fourth of that of the TAIEX index futures, the average value per transaction of the Mini-TAIEX index futures is only 5.25% [=1/4×(2.81÷13.38)] of that of the TAIEX index futures. Therefore, the investor structures of these two index futures should be different. The Mini-TAIEX futures market does attract more individual investors with fewer capitals as the TAIFEX announced to do. Because of different investor structure and longer history, the TAIEX index futures provides higher liquidity. This result triggers and supports our work to investigate the process of price discovery and information transmission for these two TAIEX index futures.

Panel B in Table 3 shows the summary statistics of prices and returns of both index futures contracts. The statistics include basic statistics, unit root test, and Engle-Granger cointegration test. The variance ratio of TAIEX futures to Mini-TAIEX futures for return series is interesting. At the signif-
Table 3 Statistics of the TAIEX and the Mini-TAIEX Stock Index Futures

<table>
<thead>
<tr>
<th></th>
<th>TAIEX Futures</th>
<th>Mini-TAIEX Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Average Daily Transaction Statistics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trading Volume</td>
<td>22,981</td>
<td>3,620</td>
</tr>
<tr>
<td># of Transaction</td>
<td>1,718</td>
<td>1,288</td>
</tr>
<tr>
<td>Volume / Transaction</td>
<td>13.38</td>
<td>2.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Panel B: Summary Statistics of 1-minute Intraday Data</strong></th>
<th>Log Price, $P_S$</th>
<th>Return, $R_{S,t}$</th>
<th>Log Price, $P_M$</th>
<th>Return, $R_{M,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation</td>
<td>7.965</td>
<td>7.964</td>
<td>7.965</td>
<td>7.964</td>
</tr>
<tr>
<td>Mean</td>
<td>8.689758</td>
<td>7.54E-06</td>
<td>8.689715</td>
<td>7.30E-06</td>
</tr>
<tr>
<td>Maximum</td>
<td>8.738735</td>
<td>0.017629</td>
<td>8.739216</td>
<td>0.017483</td>
</tr>
<tr>
<td>Minimum</td>
<td>8.610684</td>
<td>-0.013727</td>
<td>8.610866</td>
<td>-0.011390</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>0.035142</td>
<td>0.000875</td>
<td>0.035132</td>
<td>0.000957</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.685061</td>
<td>2.089328</td>
<td>-0.686184</td>
<td>1.375926</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.055264</td>
<td>70.63825</td>
<td>2.050360</td>
<td>41.75149</td>
</tr>
<tr>
<td>Unit Root Test: ADF$^a$</td>
<td>-1.12723</td>
<td>-63.10959*</td>
<td>-1.79279</td>
<td>-88.84899*</td>
</tr>
<tr>
<td>E-G Cointegration$^b$</td>
<td>-10.514*</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sample period: February 20, 2002 to March 29, 2002

$^a$ Critical value of ADF to test the hypothesis of unit root at 1% significance level is –3.4341.

$^b$ Error term of E-G cointegration test = $P_S - P_M$. Critical value at 1% significance level is –3.73.

$^c$ $R_{S,t} = \Delta P_{S,t} = P_{S,t} - P_{S,t-1}$, Critical value at 1% significance level is –3.73.

* Significance at the 1% level

At the 1% level of significance, the return variances of the TAIEX (0.000875) and the Mini-TAIEX (0.00957) index futures are significantly different (F value = 1.196). In other words, the Mini-TAIEX futures market is more volatile than the TAIEX futures market. Besides, following MacKinlay and Ramaswamy [15] and Iihara et al. [9], if the Mini-TAIEX futures prices are as informative as the TAIEX prices, the variance ratio of TAIEX futures to Mini-TAIEX futures should be close to 1. If the Mini-TAIEX futures market is noisier than the TAIEX futures market, the variance ratio must be smaller than one. Since the variance ratio is 0.914, the TAIEX index futures prices are less noisy than those of the Mini-TAIEX futures prices.
The ADF statistics of $P_X$ and $P_M$ are not significant which shows that each log series of prices has a unit root. However, $R_{X,t}$ and $R_{M,t}$, the first differences of log price series, are stationary. That is, both price series are I(1) variables. The Engle-Granger test is used to examine whether those two I(1) series are cointegrated or not. We find that there exists a linear combination of $P_X$ and $P_M$ making the residuals $Z_t = (P_{X,t} - P_{M,t})$ stationary. In other words, the prices of the Mini-TAIEX and the TAIEX stock index futures are cointegrated, and therefore both markets exhibit a stable long-run equilibrium relationship. This cointegration conclusion is useful for later analyses.

4. Methodology

4.1 Vector Error Correction Model

Because the price series are cointegrated, we use the vector error correction model (VECM) to investigate the lead-lag relationship of price discovery. The VECM employed in this study is illustrated as the following:

$$
\Delta P_t = \alpha Z_{t-1} + \sum_{i=1}^{r} B_i \Delta P_{t-i} + e_t
$$

where $\Delta P = (\Delta P_{X,t}, \Delta P_{M,t})'$ is the vector of the TAIEX and the Mini-TAIEX logarithm prices, $\alpha = (\alpha_X', \alpha_M')'$ is the error correction coefficient vector with negative $\alpha_X$ and positive $\alpha_M$, $Z_{t-1} = \beta P_{t-1} = P_{X,t-1} - P_{M,t-1}$ and $\beta = (1, -1)'$ is the cointegration vector, $B_i = (b_{XX,i}, b_{XM,i}, b_{MX,i}, b_{MM,i})'$ is a $2 \times 2$ matrix of parameters, and $e_t = (e_{X,t}', e_{M,t}')'$ is a vector of white noise innovations. For later use we define $\alpha Z_{t-1} = \alpha^\prime \beta^\prime P_t = \Pi P_t$. Since $P_{X,t}$ and $P_{M,t}$ are logarithm prices, $\Delta P_t$ represents the return vector of $(R_{X,t}, R_{M,t})'$. Because the TAIEX and the Mini-TAIEX stock index futures are substitutes for each other in terms of contract specifications and the futures market in Taiwan is competitive, even though there does exist lead-lag relationship between the prices of these two index futures, we do not think that the lag time will be too long. Therefore, we set the order of lag to be 4 in our work.

The error correction term $Z_{t-1}$ means the deviation from the “long-run” cointegration equilibrium in last period. This model interprets that the change in $P_{X,t}$ and $P_{M,t}$ is due to “short-run” effects from past $\Delta P_{X,t}$ and $\Delta P_{M,t}$,
and measures the adjustment speed to the “long-run” equilibrium. If the coefficient of error correction term of $P_{X,t}$ ($P_{M,t}$) equation is small, $P_{X,t}$ ($P_{M,t}$) has little tendency to adjust to correct a disequilibrium situation. That is, most of the adjustments will be done by $P_{M,t}$ ($P_{X,t}$), and the TAIEX (Mini-TAIEX) index futures plays an important role in price discovery. Besides, because the definition of $Z_{t-1}$ is $P_{X,t-1} - P_{M,t-1}$, $P_{M,t}$ should increase and $P_{X,t}$ should decrease to bring the price relationship back to the long-run equilibrium when $Z_{t-1} > 0$. This is the reason why we expect these two parameters in the error correction coefficient vector $\alpha = (\alpha_X, \alpha_M)'$ to be negative and positive, respectively.

The relationship of Granger causality between the TAIEX and the Mini-TAIEX index futures is determined by the coefficient of $b_{XM,i}$ and $b_{MX,i}$ which measure the short-run effects as captures by lagged returns. In other words, $b_{XM,i}$ and $b_{MX,i}$ serve the role of identifying the direction of the causal relationship. If some coefficients of $b_{XM,i}$ are non-zero, we say there is a causality relationship from the Mini-TAIEX futures to the TAIEX futures. If not all coefficients of $b_{MX,i}$ are zero, we can find the direction of information transmission from the TAIEX futures to the Mini-TAIEX futures. A joint test of block exogeneity on the coefficients of $b_{XM,i}$ ($b_{MX,i}$) being significantly different from zero is employed to test for the Granger causality between these two futures.

4.2 Gonzalo and Granger Information Shares

Based on the framework of Stock and Watson [23], Gonzalo and Granger [8] propose that, in a cointegrated system, the vector of market prices can be decomposed into a permanent component and a transitory component:

$$P_t = A_1 f_t + A_2 Z_t$$

(2)

where $m =$ the number of price series, $r =$ the rank of cointegration, $k = m - r$, $f_t$ is the common factor, $Z_t$ is the vector of error correction term, $A_1 f_t$ is the permanent component of I(1), and $A_2 Z_t$ is the I(0) transitory component that does not have a permanent effect on $P_t$. Because $f_t$ is common to all price series, we can treat $f_t$ as an implicit efficient price.

To investigate the price discovery mechanism, Gonzalo and Granger [8] focus on the contribution of each market to the determination of the implicit efficient price. They prove that the common factor can be represented as a linear combination of the price series:
where $\alpha_\perp = (\gamma_1, \gamma_2, ..., \gamma_m)'$ is a $m \times 1$ weighting vector. This definition of common factor is equivalent to a weighted average price. By imposing that the transitory component of error correction term has no long-run impact on $P_t$, they also demonstrate that the weighting vector $\alpha_\perp$ is orthogonal to the error correction coefficient vector $\alpha$ in a VECM system.

Therefore, in our bivariate system, the common factor $f_t$ is defined as the sum of $\gamma_X P_X$ and $\gamma_M P_M$. By using the result of the VECM and normalizing these weights, i.e.,

$$\gamma_X = \frac{\alpha_M}{\alpha_X + \alpha_M} \tag{4.1}$$

$$\gamma_M = \frac{\alpha_X}{\alpha_X + \alpha_M} \tag{4.2}$$

$\gamma_X$ and $\gamma_M$ become useful measures of the contribution of the TAIEX and the Mini-TAIEX futures to the common factor and, hence, to information transmission and price discovery.

### 4.3 Generalized Impulse Response Functions and Variance Decompositions

In the multivariate time series system of Sims [22], traditional dynamic analysis of vector autoregressive (VAR) models is often carried out using the orthogonalized impulse responses, where the shocks to the VAR model are orthogonalized using the Cholesky factorization. The impulse response function can measure the time profile of the effect of shocks on the futures values of a dynamic system. However, the results of traditional impulse response may differ significantly depending on the ordering of the series in the VAR system. In other words, different ordering of the price series will cause different variance decomposition results. It means that the conventional impulse response analysis can only report the range of the effect of one unit (standard error) shock of one market on the other markets.

To avoid the problem of ordering, Koop et al. [12] and Pesaran and Shin [18] propose the generalized impulse response function (GI), which is invariant to the ordering of the variables. The technical content of the GI analysis is put in the Appendix. The GI analysis gives the exact information shares for each market in the interactive system and, hence, we can investigate the information transmission among markets. By employing this methodology of GI analysis, Lin and Hsu [14] investigates the cross-market
linkage between Singapore and Taiwan futures markets for the pre- and post-participation of qualified foreign institutional investors (QFIIs) in the Taiwanese futures market.

5. Empirical Results

5.1 Information Transmission and Information Share

Panels A, B, and C in Table 4 show the results of the VECM, Granger causality, and Gonzalo-Granger information share, respectively. The coefficients of error correction term for the TAIEX and the Mini-TAIEX equations are -0.000002 and 0.25068, respectively. Because the definition of error correction term in this study is $Z_t = P_{X,t} - P_{M,t}$, when the price relationship between the TAIEX and the Mini-TAIEX stock index futures deviates away from the long-run equilibrium, both of them have the right direction of response to bring the price relationship back to equilibrium. However, only the coefficient of the Mini-TAIEX equation is statistically significant and the figure of the TAIEX equation is economically meaningless. Therefore, when the cointegration relationship is disturbed by the arrival of information, the TAIEX index futures does not adjust to departures from equilibrium and all the adjustments are accomplished through the changes of the Mini-TAIEX futures contract. Since the Mini-TAIEX futures use the TAIEX futures as the benchmark of equilibrium price, from the viewpoint of long term cointegration relationship, the TAIEX futures leads the Mini-TAIEX futures in price discovery.

The coefficients of lagged returns show that the TAIEX index futures leads the Mini-TAIEX index futures by at least four minutes (lags), while the Mini-TAIEX only leads by one minute and the magnitude is relatively smaller. The results of Granger causality in Panel B indicate that the price movements of the TAIEX index futures Granger cause those of the Mini-TAIEX index futures. However, the reverse causality from the Mini-TAIEX to the TAIEX is not significant at the 1% level. It is clear that the lead-lag relationship between the TAIEX and the Mini-TAIEX index futures are not bi-directional. The TAIEX index futures does transmit more information than the Mini-TAIEX market.

The more important role a market plays in price discovery, the more contribution it has to the determination of implicit efficient price (common factor). With the use of the estimated coefficients of error correction term in
Table 4: Results of the VECM, Granger Causality, and Information Share

<table>
<thead>
<tr>
<th>Variables</th>
<th>Coefficients (TAIEX, $P_X$)</th>
<th>Coefficients (Mini-TAIEX, $P_M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{t-1}$</td>
<td>-0.000002</td>
<td>0.250680*</td>
</tr>
<tr>
<td>$\Delta P_{X_{t-1}}$</td>
<td>0.028593*</td>
<td>0.501620*</td>
</tr>
<tr>
<td>$\Delta P_{X_{t-2}}$</td>
<td>-0.076159*</td>
<td>0.307508*</td>
</tr>
<tr>
<td>$\Delta P_{X_{t-3}}$</td>
<td>-0.041250**</td>
<td>0.188638*</td>
</tr>
<tr>
<td>$\Delta P_{X_{t-4}}$</td>
<td>-0.012162</td>
<td>0.110353*</td>
</tr>
<tr>
<td>$\Delta P_{M_{t-1}}$</td>
<td>0.059801*</td>
<td>-0.346316*</td>
</tr>
<tr>
<td>$\Delta P_{M_{t-2}}$</td>
<td>0.036762</td>
<td>-0.242961*</td>
</tr>
<tr>
<td>$\Delta P_{M_{t-3}}$</td>
<td>0.030384</td>
<td>-0.128929*</td>
</tr>
<tr>
<td>$\Delta P_{M_{t-4}}$</td>
<td>0.013467</td>
<td>-0.075392*</td>
</tr>
</tbody>
</table>

Panel B: Granger Causality

H0: $b_{XM,i} = 0$ 2.24954
H0: $b_{MX,i} = 0$ 168.138*

Panel C: Gonzalo-Granger Information Share

$\gamma_X = 99.9929\%$ 0.0008%

a. The VECM system is as follows:

$$\Delta P_{X_{t-1}} = a_X Z_{t-1} + \sum_{j=1}^{d} b_{XX,j} \Delta P_{X_{t-j}} + \sum_{j=1}^{d} b_{XM,j} \Delta P_{M_{t-j}} + e_{X,t}$$

$$\Delta P_{M_{t-1}} = a_M Z_{t-1} + \sum_{j=1}^{d} b_{XM,j} \Delta P_{X_{t-j}} + \sum_{j=1}^{d} b_{MM,j} \Delta P_{M_{t-j}} + e_{M,t}$$

where $Z_{t-1} = P_{X_{t-1}} - P_{M_{t-1}}$.

b. Granger causality test figures are F-statistics

* Significance at 1% level  ** Significance at 5% level

VECM, the Gonzalo-Granger information shares for both markets are presented in Panel C. The value of $\gamma_X$ represents that the TAIEX futures market contributes almost 100% of the common factor, and, hence, the implicit efficient price is totally dependent on the price of the TAIEX index futures. In sum, the TAIEX index futures plays a significantly important role in price discovery and the direction of information transmission is from the TAIEX to the Mini-TAIEX, not bi-directional.

5.2 Generalized Impulse Response Function and Variance Decomposition

The results of generalized impulse response functions are presented in Figure 1. The dotted (solid) line shows the generalized impulse responses of $\Delta P_M (\Delta P_X)$ to one unit (standard error) shock of $\Delta P_X (\Delta P_M)$. By visualizing the GI functions, it is easy to see that these lines are telling two different stories. The responses of the Mini-TAIEX futures to the shock of the TAIEX futures (the dotted line) at horizon 1 and 2 are 0.000327 and 0.000063, respectively. Although the effects at horizon 3 and 4 become smaller,
however, at horizon 5 and 6, we can still observe some changes in the line shape. And finally, the effects tend to die out after about horizon 7. The situation of the solid line is different. After $\Delta P_X$ makes a minor response of only 0.000066 to one unit shock of $\Delta P_M$ at horizon 1, the effects of shock are close to zero right after horizon 2. While the impact of $\Delta P_M$ shock on the return of the TAIEX index futures is smaller and lasts for only 1 period, the price movements of the TAIEX index futures have larger and persistent effects on those of the Mini-TAIEX index futures more and longer.

Table 5 presents the selected results of the generalized forecast error variance decompositions for each contract. Panel A (B) shows the contributions of the TAIEX (Mini-TAIEX) index futures to the variance in $\Delta P_X$ and $\Delta P_M$ at some selected horizons. While the $\Delta P_X$ shock explains 80% of the variance in $\Delta P_M$, the $\Delta P_M$ innovation only accounts for around 68% of the variability of $P_X$. Based on the results of GI analysis, we confirm again that the TAIEX stock index futures dominates the Mini-TAIEX stock index futures in price discovery, and, hence, transmit more information.

![Figure 1 Generalized Impulse Responses to One Unit Shock](image-url)
Table 5 Selected Generalized Forecast Error Variance Decompositions

<table>
<thead>
<tr>
<th>Horizon</th>
<th>$\Delta P_X$</th>
<th>$\Delta P_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: TAIEX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.6324</td>
</tr>
<tr>
<td>1</td>
<td>0.9365</td>
<td>0.7666</td>
</tr>
<tr>
<td>2</td>
<td>0.9638</td>
<td>0.8079</td>
</tr>
<tr>
<td>5</td>
<td>0.9582</td>
<td>0.8023</td>
</tr>
<tr>
<td>10</td>
<td>0.9611</td>
<td>0.8065</td>
</tr>
<tr>
<td>Panel B: Mini-TAIEX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.7198</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.6538</td>
<td>0.9266</td>
</tr>
<tr>
<td>2</td>
<td>0.6829</td>
<td>0.9563</td>
</tr>
<tr>
<td>5</td>
<td>0.6767</td>
<td>0.9507</td>
</tr>
<tr>
<td>10</td>
<td>0.6798</td>
<td>0.9549</td>
</tr>
</tbody>
</table>

6. Summary and Conclusion

Many empirical studies have reported significant lead-lag relationships between or among highly related financial assets. Although researchers argue that the differences of transaction costs, trading mechanism, market liquidity, and among other factors are the possible explanations, it is not easy to clarify the contributions of differences, which simultaneously exist among those highly related financial securities, to the lead-lag relationship.

Basically, the TAIEX and the Mini-TAIEX stock index futures are identical securities in terms of contract specifications and trading mechanism, except for their contract sizes. However, the TAIEX futures market provides higher liquidity. By examining the information transmission process, we can identify whether market liquidity contributes to the lead-lag relationship.

Empirical results support that there is a significant information transmission from the TAIEX to the Mini-TAIEX stock index futures. The TAIEX does not adjust to departures from equilibrium and all the adjustments are accomplished through the changes of the Mini-TAIEX index futures contract. The lagged returns of the Mini-TAIEX index futures can not explain the price change of the TAIEX index futures. Gonzalo-Granger information share shows that the TAIEX futures market contributes almost 100% of the common factor (implicit efficient price). The results of GI analysis also indicate that the changes in the TAIEX price have stronger and
longer impacts on the Mini-TAIEX index futures market. In sum, the TAIEX index futures play the major role in price discovery and, because they share the same trading mechanism and similar contract specifications, we conclude that the higher market liquidity in the TAIEX index futures is the underlying factor.

References


in nonlinear multivariate models. *Journal of Econometrics* 74 119-147.


Appendix

In this section, we briefly introduce the methodology of generalized impulse response functions and variance decompositions.

Consider the following VAR(\(p\)) model:

\[
P_t = \sum_{i=1}^{p} \Phi_i P_{t-i} + e_t
\]

where \(P_t\) is an \(m\times1\) vector of price series and \(e_t\) is the vector of independent identically distributed errors with mean zero and variance \(\Sigma\). Under the assumption of stationary \(P_t\), equation (5) can be transformed into an infinite moving average VMA(\(\infty\)) model.

\[
P_t = \sum_{i=0}^{\infty} A_i e_{t-i}
\]

where the coefficient matrices \(A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + \cdots + \Phi_p A_{i-p}\), \(i = 1, 2, \ldots\), with \(A_0 = I_m\) and \(A_i = 0\) for \(i < 0\).

Koop et al. [12] define the GI as the following:

\[
GI_p(n, e_t, \Omega_{t-1}) = E \left[ P_{t+n} | e_t, \Omega_{t-1} \right] - E \left[ P_{t+n} | \Omega_{t-1} \right]
\]

where \(e_t\) is the shock vector, \(\Omega_{t-1}\) is the information set at time \(t-1\), and \(E[P_{t+n} | \cdot]\) represents the expected prices at time \(t+n\) conditioned on only the history and/or shock. For the case of equation (6),

\[
GI_p(n, e_t, \Omega_{t-1}) = A_n e_t
\]

which is independent of \(\Omega_{t-1}\). If the GI function could be scaled by \(e_t\), it reduces to \(A_n\), the similar traditional impulse response function. However, because \(e_t\) is a vector, there is no “appropriate” scale to eliminate its effect on the GI function. Hence, the GI analysis depends on the composition of shocks.

Koop et al. [12] further define the GI to be conditional on only one element at time \(t\), say the \(j\)th shock, and then integrate out the effects of the other shocks at time \(t\) given its value, \(\varepsilon_{jt}\),

\[
GI_p(n, \varepsilon_{jt}, \Omega_{t-1}) = E \left[ P_{t+n} | \varepsilon_{jt} = \varepsilon_{jt}, \Omega_{t-1} \right] - E \left[ P_{t+n} | \Omega_{t-1} \right]
\]

Assuming that \(e_t\) is jointly normally distributed and that the conditional expectation of the shocks is a linear function of \(\varepsilon_{jt}\):

\[
E \left[ e_t | \varepsilon_{jt} = \varepsilon_{jt} \right] = (\sigma_{1j}, \sigma_{2j}, \ldots, \sigma_{mj})^T \varepsilon_{jt} = \Sigma_j \varepsilon_{jt}
\]

\[
(10)
\]
where $\sigma_{jj} = \mathbb{E}[\varepsilon_{jt}^2]$ and $s_j$ is a selection vector with its $j$th element equal to unity and zeros elsewhere.

The generalized impulse response of the effect of a shock in the $j$th equation at time $t$ on $P_{t+n}$ is then given by

$$GI_p(n, \varepsilon_{jt}, \Omega_{t-1}) = A_n \sum_j \sigma^{-1}_{jj} \varepsilon_{jt} = \begin{pmatrix} A_n \sum_j \frac{\varepsilon_{jt}}{\sigma_{jj}} \end{pmatrix} \begin{pmatrix} \sigma_{jj} \end{pmatrix} \begin{pmatrix} \varepsilon_{jt} \end{pmatrix} \quad (11)$$

By setting $\varepsilon_{jt} = \sqrt{\sigma_{jj}}$, we obtain the effect of one ‘unit’ (standard error) shock to the $j$th equation on the expected $P_{t+n}$ by

$$\Psi^GJ_{j} (n) = \begin{pmatrix} A_n \sum_j \frac{\varepsilon_{jt}}{\sqrt{\sigma_{jj}}} \end{pmatrix} \quad (12)$$

As we can see, the GI are not affected by reordering of the variables in $P_t$.

Based on the GI analysis, Pesaran and Shin [18] define the forecast error variance decompositions as the proportion of the n-step ahead forecast error variance of variable $i$ which is accounted for by the shocks in variable $j$.

$$\theta_{ij}^{GI} (n) = \frac{\sigma_{jj}^{-1} \sum_{q=0}^{n} (s_i A_q \sum_j)^2}{\sum_{q=0}^{n} s_i^2 A_q \sum_{q}^n A_q s_i} \quad (13)$$

While the sum of orthogonalized variance decompositions $\sum_{j=1}^{m} \theta_{ij}^O (n)$ is equal to unity, that of the GI variance decompositions $\sum_{j=1}^{m} \theta_{ij}^{GI} (n)$ is not in general. However, we can still find the relative importance for each market in price discovery and information transmission by comparing their contributions to the variances of other markets.

For a cointegration VAR model, the GI analysis can be done as the following steps. The VECM form of equation (1) can be transformed into an infinite moving average representation

$$\Delta P_t = \sum_{i=0}^{\infty} C_i \varepsilon_{t-i} \quad (14)$$

where

$$C_0 = C(1) + C^*_0 = I_m \quad (15)$$

$$C_i = C^*_i - C^*_{i-1}, \quad i = 1, 2, ... \quad (16)$$

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where \( \Pi C(I) = 0 \), \( C_i^* \) are computed by

\[
C_i^* = C_{i-1}^* \Phi_1 + C_{i-2}^* \Phi_2 + \cdots + C_{i-r-1}^* \Phi_{r+1}
\]

(17)

for \( i = 1, 2, \ldots \) with \( C_0^* = I_m - C(I) \), \( C_i^* = 0 \), \( i < 0 \), and in terms of \( \Pi \) and \( B_i \) in the VECM form the \( \Phi_i \) matrices are obtained by

\[
\Phi_i = I_m + \Pi + B_1, \quad \Phi_i = B_i - B_{i-1} \quad \text{for} \quad i = 2, 3, \ldots, r, \quad \Phi_{r+1} = -B_r
\]

(18)

Once we transform the VECM form of equation (1) to the VMA (\( \infty \)) system in equation (14), the generalized impulse response functions and the generalized forecast error variance decompositions can also be computed as the same concepts and steps from equation (7) to equation (13)\(^1\).

\(^1\) For more detailed discuss of the GI analysis and transformation of VECM to VMA see Pesaran and Pesaran [17] and Johansen [10].