Design a continuous sampling plan based on quadratic quality loss function

Chung-Ho Chen* and Chao-Yu Chou**

In this paper, we present the problem of determining the cost-optimal Type I continuous sampling plan (CSP-I plan) under Taguchi's quality loss function. Assume that the quality fluctuates between two levels according to a two-state time homogeneous Markov chain. This article proposes the heuristic method for obtaining the optimal inspection policy.

Keywords: Type I Continuous Sampling Plan (CSP-I Plan); Average Fraction Inspected (AFI); Taguchi's Quality Loss Function; Markov Process

1. Introduction

In 1943, Dodge [5] presented a continuous inspection procedure commonly referred to as Type I continuous sampling plan (CSP-I plan). The procedure of the above plan is as follows: Inspect every item until i successive items are found free of defects, and then inspect at a fraction f of the item; when a defective item is found, revert to 100% inspection and continue until again i successive items are found free of defects. All defective items found in inspection are either reworked or replaced by good items.

Dodge's [5] CSP-I plan can be used for process control to assure product quality. An assumption of the traditional CSP-1 plan is that the production process is stable. Wetherill [17] and Vander Wiel and Vardeman [16] have shown that the cost-optimal inspection policy for a CSP-1 plan with steady production process and a sum of identically determined inspection costs for each item is either no inspection or 100 percent inspection. Previous researchers (Lieberman [10], Derman et al. [4], Hillier [8], Lasater [9], Endres [6], and Sackrowitz [13]) have criticized the independent and identically distributed (iid) Bernoulli assumption of the CSP-1 plan. McShane and Turnbull [11] have presented some studies of the CSP-1 plan under Markovian manufacturing processes.

In 1986, Taguchi [15] presented the quadratic quality loss function for reducing deviation from the target value. The objective of this quality im-
Improvement method is to reduce total losses to the society. Taguchi’s [15] loss function can be used in off-line and on-line quality control. Recently, Tagaras [14], Arizono et al. [1], Derman and Ross [3], and Fink and Margavio [7] have developed new variable sampling plans based on Taguchi’s [15] loss criterion.

Chen [2] have addressed that Taguchi’s [15] quality loss function is applied in the design of attribute sampling plan. In this paper, we further present the problem of determining the cost-optimal CSP-1 plan under Taguchi’s [15] quality loss function. Assume that the quality fluctuates between two levels according to a two-state time homogeneous Markov chain. By using McShane and Turnbull’s [11] correlated input model, we can formulate the model of the expected total quality loss per item for a CSP-1 plan. Finally, the heuristic method for obtaining the optimal inspection policy is proposed.

2. Assumptions

1. The quality fluctuates between two levels according to a two-state homogeneous Markov chain.
2. Inspection is perfect.
3. The defective item inspected and reworked possesses perfect quality.
4. All items can be reworked—not scrapped.

3. Expected total quality loss model

A random variable $X$ has a Poisson distribution with the following probability mass function

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, x = 0, 1, 2, \ldots$$ (1)

The expected value and variance for the Poisson distribution are $\mu = \lambda$ and $\sigma^2 = \lambda$, respectively. For smaller-the-better type quality characteristic, $X$, the average loss per item for no inspection is

$$L_0 = kE[(X - 0)^2] = k(\sigma^2 + \mu^2) = k(\lambda + \lambda^2)$$ (2)

where $k$ = a constant called quality loss coefficient. The average loss per item for 100 percent inspection is
\[ L_1 = IC + p \cdot SC \]  

where \( SC \) = the cost to rework a defective item  
\( IC \) = the cost of inspection each item  
\( p \) = the proportion of non-conforming items found during a long-run production.

McShane and Turnbull [11] have investigated the performance of a CSP-1 plan when the production run length is short or moderate or the input process is not iid Bernoulli. According to McShane and Turnbull [11] correlated input model, some results for a CSP-1 plan with two-state time homogeneous Markov chain are as follows:

\[ AFI = f[(a+b)+b(1-f)(1-a)^{-1}(a+b-1)]/(a+b)[f+b(1-f)(1-a)^{-1}] \]  
\[ p = a[(a+b)+b(1-f)(1-a)^{-1}(a+b-1)] \]

where \( i \) = the clearance number of the 100% inspection stage for a CSP-1 plan  
\( f \) = the fraction of the items to be sampled and inspected in sampling inspection stage for a CSP-1 plan  
\( AFI \) = the average fraction inspected for a CSP-1 plan  
\( a = P[(m+1)\text{th item is defective} | \text{mth item non-defective}], \ 0 < a < 1 \)  
\( b = P[(m+1)\text{th item is non-defective} | \text{mth item defective}], \ 0 < b < 1 \).

Hence, the expected total quality loss per item for a CSP-1 plan is as follows:

\[ EC(i, f) = AFI \cdot L_1 + (1 - AFI) \cdot L_0 \]  
\[ = AFI \cdot (L_1 - L_0) + L_0 \]  
\[ = AFI \cdot (IC + p \cdot SC - L_0) + L_0 \]  

4. Solution procedure

Montgomery [12] points out that as a general rule, it is not a good idea to choose values of sampling interval \( n = 1/f \) larger than 200 for a CSP-1 plan because the production against bad quality in a continuous run of production then becomes very poor.

Hence, we can use the following heuristic method to find the parameters \((i, f)\) that satisfy the minimum \( EC(i, f) \).

Step 1. Let \( f = 0.005 \) and \( 1 \leq i \leq 5000 \). To compute the corresponding
EC(i, f) for the given a, b, k, IC, SC, and λ.

Step 2. For all the \( f = 0.005 (0.0001) 1 \), we repeat step 1 in order to compute the corresponding \( EC(i, f) \). By comparing the respective \( EC(i, f) \), we can select the most reasonable parameters \( (i, f) \) that have the minimum expected total quality loss per item.

5. Numerical example

5.1. Example 1

Suppose that \( a = 0.02 \), \( b = 0.08 \), \( IC = 1 \), and \( SC = 4 \). The estimated values of \( \lambda \) and \( k \) are \( \lambda = 2 \) and \( k = 1.2 \), respectively. By adopting the above solution procedure, we obtain the most reasonable parameters \( (i, f) \) that minimize Eq. (6) are \( (i, f) = (974, 0.005) \) with \( L_0 = 7.2 \), \( L_1 = 1.8 \), \( EC(i, f) = 1.8 \) and \( AFI = 1 \).

5.2. Example 2

Suppose that \( a = 0.02 \), \( b = 0.08 \), \( IC = 1 \), and \( SC = 4 \). The estimated values of \( \lambda \) and \( k \) are \( \lambda = 2 \) and \( k = 0.25 \), respectively. By adopting the above solution procedure, we obtain the most reasonable parameters \( (i, f) \) that minimize Eq. (6) are \( (i, f) = (1, 0.005) \) with \( L_0 = 1.5 \), \( L_1 = 3.8209 \), \( EC(i, f) = 1.5389 \) and \( AFI = 0.0168 \).

6. Conclusions

In this paper, we have presented the problem of determining the cost-optimal CSP-1 plan under Taguchi’s [15] quality loss function. This work is an extension of Vander Vei's and Vardeman’s [16] study. Supposing that the quality fluctuates between two levels according to a two-state time homogeneous Markov chain, we propose the heuristic method for obtaining the optimal inspection policy for a CSP-1 plan. Further study of direction will address the larger-the-better or nominal-the-best quality characteristic for the design of CSP-1 plan.

References


19-24.


