Valuation of R&D Projects

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Abstract

This work presents a novel option pricing method to evaluate R&D projects, an area that has received considerable attention in recent years. As the underlying assets, e.g. R&D projects, are usually non-tradable, this work examines a “twin” or “surrogate” traded asset whose price process is highly correlated with that of a non-traded underlying asset. Based on the observable market price of the surrogate asset, the proposed model can price the value of a real option underlying the non-traded R&D projects. The proposed model differs from previous ones mainly owing to its ability to deal with highly complex situations, e.g. the compound effect and the non-tradability of R&D projects, when evaluating them.

Keywords: R&D project, compound option, non-traded asset, incomplete market.

1. Introduction

Given advances in financial engineering, option pricing methods have been extensively adopted to evaluate investment projects by firms. Firm managers most widely utilize option pricing methods to analyze investment strategies and make decisions dynamically according to changes in the investment environment. The option pricing methodology can evaluate investment projects more precisely than the conventional net present value (NPV) criterion. The conventional NPV criterion always underestimates the actual value of a project under circumstances in which future management actions can increase profits or reduce losses. Myers (1984) also indicated that the NPV criterion is inadequate since under this criterion, firms seeking to establish a foothold in a market never invest in negative-NPV projects.

Among all investment projects, R&D projects, due to their long planning horizon and uncertainty, illustrate the advantages of option pricing methodologies over the conventional NPV criteria. Although substantial, the value of managerial flexibility with R&D projects is not considered in conventional methods. While describing basic characteristics of the real options approach, Perlitz, et al. (1999) shed light on problems with applications involving R&D project evaluation. Jägle (1999) developed a binomial option tree that incorporates the flexibility and risks inherent in new product development. Lee and Paxson (2001) modeled the stages of e-commerce R&D expense and the applicability of real sequential exchange option models. Lint and Pennings (1998, 2001) described option pricing model applications for setting the budget of R&D projects and applying this approach to the valuation of multimedia R&D projects at Philips Corporate Research. Brach and Paxson (2001) devised a Poisson real option model to evaluate a gene-to-drug venture. Biekpe, et al. (2001) demonstrated the feasibility of using power series expansions to obtain an analytic expression for the value of a firm’s investment opportunities and in an R&D setting to determine the investment value when two conventional stochastic processes generate cash flows. Jensen and Warren (2001) applied the Geske’s compound options model, based on a three-phase lifecycle

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of research, development and deployment, to an e-commerce case study to extract key messages for R&D managers relating to different value drivers. Hartmann and Hassan (2006) investigated the feasibility of performing real options analysis in the pharmaceutical industry. Kim, et al. (2008) developed a valuation framework for an investment project through the real options concept. That study also verified how investment lag affects investment value, as well as provided a closed-form formula for valuing real assets, including a R&D project. According to Bekkum, et al. (2009), the conditionality of investment decisions in R&D significantly impacts portfolio risk, implying that conventional diversification strategies should be reevaluated when a portfolio is constructed. Despite developing various option-based project evaluation methods, the above works are limited in that the underlying asset of R&D projects is usually non-tradable. Therefore, the model proposed in this work improves upon Geske’s compound option model by reflecting that the underlying assets of R&D projects are always non-tradable.

When a contingent claim or option whose underlying asset is non-traded is priced, the first challenge to be faced is that the market price of the non-traded underlying asset is unobservable. Therefore, deriving the option price by adopting the method of either Black and Scholes (1973) or Geske (1979) to form a no-arbitrage portfolio and then setting the return of this portfolio to be the risk-free interest rate to derive the option price seems to be impossible (Trigeorgis, 1993). Since each R&D project is always a new innovation, its market price after completion is unobservable. Consequently, determining the market value of option value based on this asset is difficult. Numerous researchers such as Trigeorgis (1993), Pindyck (1993) and Dixit and Pindyck (1993) have suggested various methods to solve this problem. The most well-known one is the “spanning method”, which duplicates a non-traded asset. The “spanning method” to find the value of the non-traded asset that underlies an option depends on the finding of a “surrogate” asset of the original non-traded underlying asset, to form a no-arbitrage portfolio. Since the asset is only a “surrogate” asset, it cannot perfectly substitute for the original non-traded asset. Hence, forming the no-arbitrage portfolio using the “surrogate” asset and the option is an “approximate hedge” with respect to the non-traded asset. The similarity between the surrogate asset and the non-traded asset determines the effectiveness of the spanning method.

Given the above developments, this work presents a novel compound option model constructed in an incomplete market to comply with the non-tradable characteristics of R&D projects. The compound option is incorporated to capture the multistage nature of the activity. The value of an R&D investment is not determined mainly by the cash flows coming from the initial investment but, rather, by the future investment opportunities provided by the original investment. Therefore, using the compound option to handle this compound effect is more feasible than using the general Black-Scholes method. To capture the non-tradability of R&D projects, this work draws upon the results of Cochrane and Saà-Requejo (2000) to resolve this incomplete market problem.

Cochrane and Saà-Requejo (2000) proposed recently a method, called the “Good-deal bound” method, to evaluate the contingent claim payoff whose underlying asset is a non-traded or thinly traded asset. Their method measures the attractiveness of a payoff by its Sharpe ratio, and uses the Hansen-Jagannathan inequality to find as corresponding restriction a bound on the variance of its pricing kernel. The pricing results of this method can be a pricing bound rather than a single price. This bound is called the “Good-deal bound.” Since an exact solution for options underlying non-tradable assets cannot be found, the work of Cochrane and Saà-Requejo (2000) is used to find “good-deal bounds” for an option on an R&D project.
The rest of this paper is organized as follows. Section 2 introduced the proposed model for evaluating R&D projects. Section 3 then provides examples of underlying options on a multistage R&D project to apply the proposed model. Next, Section 4 discusses the hedging strategies for the proposed model. Additionally, Section 5 presents difficulties encountered when evaluating an R&D project. Conclusions are finally drawn in Section 6, along with recommendations for future research.

2. The Model

This section introduces a novel method to evaluate a compound option with a non-tradable underlying asset. The model is developed based on the fact that nearly all R&D projects include multiple-stage investments, in which each installment is an option on the subsequent stage. Few projects can be finished in a single stage, and the investment cost can be determined in the beginning of the project. Hence, features of the compound options are introduced before constructing the proposed model.

2.1 Compound Option

Following the work of Geske (1979), the compound option has been extensively used by researchers of corporate finance. Numerous corporate investment opportunities comprise several stages that are viewed as options and have the characteristics of compound options. Four compound options, generalized by Rubinstein (1991), are commonly used in financial markets; they are calls on calls, puts on puts, puts on calls and calls on puts.

Let \( S \) be the stock price of a firm and \( U \) be the value of the firm. Neither pays a dividend before maturity. The stochastic processes of \( S \) and \( U \) are assumed to be described as follows.

\[
\frac{dS}{S} = \mu_i dt + \sigma_i dz, \quad i \in \{U, S\}
\]

(1)

where \( \mu_i \) are the instantaneous expected rates of return of the firm (when \( i = U \)) and stock (when \( i = S \)) per unit time, and \( \sigma_i \) are the instantaneous standard deviations of the return of the firm (when \( i = U \)) and stock (when \( i = S \)) per unit time.

Define \( CC_i(U_i, K_U, K_S) \) as the value of the compound option at time \( t \). \( K_S \) is the exercise price of the compound option and \( K_U \) is the exercise price of the underlying option. In the method of Black-Scholes (1973) for establishing a riskless portfolio and setting the return of this portfolio to be the riskless interest rate, \( CC_i(U_i, K_U, K_S) \) must satisfy the following partial differential equation.

\[
rCC - \frac{\partial CC}{\partial t} - rU \frac{\partial CC}{\partial U} - \frac{1}{2} \sigma_i^2 U^2 \frac{\partial^2 CC}{\partial U^2} = 0,
\]

(2)

with the boundary condition,

\[
CC_i(U_i, K_U, K_S) = \text{Max}(S_i - K_S, 0).
\]

(3)

The boundary condition given by Eq. (3) poses the problem that the stochastic variable that determines the option’s value in Eq. (2) is not the stock price \( S \), which is in the boundary condition; rather, it is the firm’s value \( U \). The firm is treated as the underlying asset, and the
stock value is the option on it. From Itô’s lemma, the relationship between $S$ and $U$ can be derived as follows.

$$rS - \frac{\partial S}{\partial t} - rU - \frac{1}{2} \sigma_U^2 U^2 \frac{\partial^2 S}{\partial U^2} = 0,$$

and the boundary condition is established again as follows.

$$S_T = \max\left(U_T - K_U, 0\right).$$

Hence, the current value of a European call on a call option can be derived by solving Eq. (2) and Eq. (4).

$$CC_0(U_0, K_U, K_S) = U_0 N_2(x, y; \rho) - K_U e^{-rT} N_2\left(x - \sigma_U \sqrt{T}, y - \sigma_U \sqrt{T}; \rho\right)$$

$$- K_S e^{-rT} N\left(x - \sigma_U \sqrt{T}\right),$$

where

$$x = \frac{\ln \frac{U_0}{U_{T-t}} + \left(r + \frac{1}{2} \sigma_U^2\right) t}{\sigma_U \sqrt{T}}, \quad y = \frac{\ln \frac{U_0}{K_U} + \left(r + \frac{1}{2} \sigma_U^2\right) T}{\sigma_U \sqrt{T}} \quad \text{and} \quad \rho = \frac{t}{T}.$$
where \( r \) represents the risk-free rate and \( \sigma_{YZ} \), \( \sigma_{YW} \) represent the volatilities of the hedgeable and non-hedgeable parts of the non-traded basis asset \( Y_i \).

After the stochastic process of the non-traded underlying asset has been defined, another stochastic process, which represents the proxy asset is defined as follows.

\[
\frac{d\alpha_t}{\alpha_t} = \mu_d dt + \sigma_d dz,
\]

where \( \mu_d \) is the expected rate of return of \( \alpha_t \) and \( \sigma_d \) is the volatility of \( \alpha_t \).

The correlation between \( Y_i \) and \( \alpha_t \) is important. A higher correlation of \( Y_i \) with \( \alpha_t \) corresponds to a narrower price range that can be found. The difference between \( Y_i \) and \( \alpha_t \), is that the stochastic process of \( Y_i \) has one more diffusion term (\( dw \)) than \( \alpha_t \). This additional diffusion term represents the non-hedgeable part of \( Y_i \) when \( \alpha_t \) is substituted for \( Y_i \).

When the market is complete and no friction exists, the discount factor can be expressed as follows.

\[
\frac{dM_t}{M_t} = -r_t dt - h_d dB_t,
\]

where \( r_t \) is the instantaneous risk-free rate; \( h_d \) is the market price of risk, and \( B_t \) is a Brownian motion.

The “no arbitrage” method that was used by Black-Scholes (1973) can be employed to find the option value only when a duplicate portfolio can be formed. Because the underlying asset is non-traded, the no-arbitrage method breaks down and some other methods, such as the “spanning method” must be adopted to overcome this difficulty. Table 1 shows theories for application in an incomplete market, using which a model can be constructed to solve the issue of non-tradability.

This article employs the spanning method proposed by Cochrane and Saà-Requejo (2000) to overcome the issue of non-tradability when the real option that is based on an R&D project is evaluated. To employ this method, positive and volatility constraints are added to the optimization problem to eliminate any arbitrage opportunity and any deal with an unreasonably high Sharpe ratio to determine the reasonable price bound. This bound is called the good-deal bound. Define \( \tilde{C}_0 (\gamma_0, K_Y, K_S) \) as the good-deal lower bound of the value of the compound option at time 0; it can be derived by solving the following minimization problem:

\[
\tilde{C}_0 (\gamma_0, K_Y, K_S) = \min_M E_0^P \left[ \frac{\tilde{M}_t}{M_0} \max (S_t - K_S, 0) \right],
\]

(10)
\[ S_t = E^\rho_t \left[ \frac{\tilde{M}_t}{M_t} \text{Max} \left( Y_t - K_y, 0 \right) \right] \]
\[ \text{s.t.} \quad \tilde{M}_t > 0; \quad \frac{1}{dt} \left[ E^\rho_t \left( \frac{d\tilde{M}_t}{M_t} \right)^2 \right] \leq A^2; \quad 0 \leq t \leq T. \]

where \( A \) denotes the volatility constraint; \( K_y \) represents the strike price of the compound option with expiration \( t \), and \( K_y \) denotes the strike price of the underlying option with expiration \( T \). The upper bound for the price of the compound call option can be derived by solving the corresponding maximization problem in the same manner.

The pricing kernel in the incomplete market environment can be expressed as follows.
\[ \frac{d\tilde{M}_t}{M_t} = -r dt - h_\alpha dz + \sqrt{A^2 - h_\alpha^2} d\omega, \]

where \( h_\alpha \) represents the Sharpe ratio of the tradable asset.

Determining the price of the first option as a minimization problem in Eq. (10) yields the following value.
\[ S_t = Y_e^{(T-t)} N \left( Z_t \right) - K_y e^{-r(T-t)} N \left( Z_t - \sigma_y \sqrt{T-t} \right), \]

where \( r \) is a constant risk-free rate; \( Z_t = \ln \left( \frac{Y_t}{K_y} \right) + \left( r + \xi + \frac{1}{2} \sigma_y^2 \right) (T-t) \), and \( N(\cdot) \) is the left tail of the normal distribution.

After the first option is derived, the value of the right to invest in further stages during the evolution of the R&D project is obtained. Treating the first option as the underlying asset allows the value of the R&D project to be obtained by applying the following compound option model.
\[ \tilde{C}_0 \left( Y_0, K_y, K_y \right) = Y_0 e^{\xi T} N_2 \left( x', y', \rho \right) - K_y e^{-rT} N_2 \left( x' - \sigma_y \sqrt{t}, y' - \sigma_y \sqrt{T}, \rho \right) \]
\[ -K_y e^{-rT} N \left( x' - \sigma_y \sqrt{t} \right), \]

where
\[ x' = \frac{\ln \left( \frac{Y_0}{Y_t} \right) + \left( r + \xi + \frac{1}{2} \sigma_y^2 \right) t}{\sigma_y \sqrt{t}}, \]
\[ y' = \frac{\ln \left( \frac{Y_0}{K_y} \right) + \left( r + \xi + \frac{1}{2} \sigma_y^2 \right) T}{\sigma_y \sqrt{T}}, \]
\[ \sigma_y^2 \equiv E_t \left[ \frac{dY_t}{Y_t} \right]^2 = \sigma_{y_1}^2 + \sigma_{Y_2}^2, \]
\[ \xi \equiv \left[ h_\gamma - h_\alpha \left( \rho_1 - a \frac{A^2}{h_\alpha^2} - 1 - \rho_1^2 \right) \right] \sigma_y, \]
\[ h_\alpha = \frac{\mu_y - r}{\sigma_y}, \]
\[ h_\gamma = \frac{\mu_y - r}{\sigma_y}, \]
\[ \rho_1 = \text{corr} \left( \frac{dY}{Y}, \frac{d\alpha}{\alpha} \right) = \frac{\sigma_y}{\sigma_\gamma}, \]
\[ a = \begin{cases} +1, & \text{upper bound} \\ -1, & \text{lower bound} \end{cases} \]

\( Y_{T-t} \) is the value of \( Y_{T-t} \) for which the underlying option price equals \( K_\xi \).

Comparing Eq. (12) with Eq. (6) reveals that Eq. (12) has one additional parameter \( \zeta \). The parameter \( \zeta \) captures the effect of non-tradability when the option is priced in an incomplete market. When \( \zeta = 0 \), Eq. (12) degenerates to Eq. (6), indicating Eq. (12) is a generalized version of Geske’s formula in Eq. (6). \( \zeta \) resembles the “incomplete market spread,” which discounts the option price when the assets exhibits non-tradability and the market exhibits incompleteness.

### Table 1. Summary of the incomplete market theory.

<table>
<thead>
<tr>
<th>Studies</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cochrane and Saà-Requejo (2000)</td>
<td>“discount factor” asset pricing method to evaluate uncertain payoffs</td>
</tr>
<tr>
<td>Bernardo and Ledoit (2000)</td>
<td>investments with high ratios of gains to losses</td>
</tr>
<tr>
<td>Henderson (2002)</td>
<td>utility maximization and duality methods to obtain a series approximation to the optimal hedge and a reservation price using power and exponential utility</td>
</tr>
<tr>
<td>Henderson and Hobson (2002)</td>
<td>utility maximization and duality methods to obtain a series approximation to the optimal hedge and a reservation price using power and exponential utility</td>
</tr>
<tr>
<td>Henderson and Hobson (2004)</td>
<td>utility maximization and duality methods to obtain a series approximation to the optimal hedge and a reservation price using power and exponential utility</td>
</tr>
<tr>
<td>Heath, Platen, and Schweizer (2001)</td>
<td>comparative theoretical and numerical results concerning risks, values, and hedging strategies for local risk-minimization versus mean-variance hedging in a class of stochastic volatility models</td>
</tr>
<tr>
<td>Carr, Geman, and Madan (2001)</td>
<td>a new method for positioning, pricing, and hedging in incomplete markets that bridges standard arbitrage pricing and expected utility maximization</td>
</tr>
<tr>
<td>Luenberger (2001, 2002)</td>
<td>the projection perspective of the Capital Asset Pricing Model, suggesting a new pricing formula, called the Correlation Pricing Formula (CPF) to price the non-tradable basis assets</td>
</tr>
<tr>
<td>Dixit and Pindyck (1993), Pindyck (1993), Trigeorgis (1993)</td>
<td>use spanning assets to track or span the risk in the underlying asset that is not directly traded in the market</td>
</tr>
</tbody>
</table>

### 3. Applications

This section describes a potential application of the proposed model through an example. This example involves the evolution of a new drug. As is well known developing a drug or a
technology is extremely difficult owing to the high uncertainty of the development process. For instance, even drug components identified in a laboratory must undergo a series of clinical trials. In the United States, for example, from development to commercialization, a drug must pass through discovery, pre-clinical trials, Phase I clinical trials, Phase II clinical trials, Phase III clinical trials, FDA (Food and Drug Administration) registration and review, and introduction/distribution. Figure 1 shows a flow chart of the data collected from GENELABS. Figure 2 shows the research and development process in new drug innovation. The process in Fig. 1 is divided into three stages: Drug discovery, clinical development, registration FDA review, and market introduction. At time $0$, an initial investment is made to search for active substances. In this stage, chemists and biologists must expend a considerable amount of effort in developing a concept for synthesizing new molecular entities. After completing the screening process, the company can decide whether to make a further investment of $K_T$ to clinically test this new drug at time $t_1$. For a company deciding to invest in $K_T$ for clinical development, the new drug undergoes a sequence of tests.

A. Pre-clinical

In this stage, work with the potential drug, done exclusively by researchers as opposed to clinical development individuals and nonhuman experimentation, has led scientists to assume that the substance may be useful for a specific application in humans.

### Table

<table>
<thead>
<tr>
<th>Drug Discovery</th>
<th>Preclinical Development</th>
<th>Clinical Development</th>
<th>Registration FDA Review</th>
<th>Introduction/Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Synthesis</em></td>
<td><em>Pharmacokinetic</em></td>
<td><em>Phase II</em></td>
<td><em>Evidence of safety &amp; substantial evidence of efficacy</em></td>
<td><em>Drug monitoring/ adverse reaction reporting</em></td>
</tr>
<tr>
<td><em>Biological Tests</em></td>
<td><em>Toxicity</em></td>
<td><em>Phase III / NDA Submission</em></td>
<td><em>Cost/benefit, efficacy &amp; outcome research studies</em></td>
<td><em>Post-marketing surveillance studies</em></td>
</tr>
<tr>
<td><em>Screening</em></td>
<td><em>20-50 Subjects</em></td>
<td><em>Efficacy</em></td>
<td><em>1000-3000 patients</em></td>
<td><em>Special studies</em></td>
</tr>
<tr>
<td></td>
<td><em>Toxicological Studies</em></td>
<td><em>Dose-ranging</em></td>
<td><em>1000-3000 patients</em></td>
<td><em>FDA Advisory Committee Meeting</em></td>
</tr>
<tr>
<td></td>
<td><em>Acute, subacute, chronic, reproduction, carcinogenicity studies</em></td>
<td><em>Further tolerability</em></td>
<td><em>Production, QC of production</em></td>
<td><em>New formulation development if needed</em></td>
</tr>
<tr>
<td></td>
<td>Pharmacokinetic &amp; Pharmacology Studies</td>
<td><em>100-300 patients</em></td>
<td><em>Synergistic studies</em></td>
<td><em>Line extension studies</em></td>
</tr>
</tbody>
</table>

Figure 1. New drug development process.
B. Clinical trial

This trial usually includes three phases.

(a) Phase I

This stage assesses the toxicity and dosage of the drug in humans. Clearly, the drug has sufficient toxicity effects in humans such that development should not proceed.

(b) Phase II

The trials in Phase II are significantly larger in scale than in phase I trials. This phase verifies the biological effectiveness of the drug, profiles the side effects, and obtains dosage information.

(c) Phase III

While involving large-scale patient trials to demonstrate its efficacy, this phase is carefully designed for FDA drug approval.

After all clinical tests are finished at time $t_2$, a company must decide whether to invest in $K_S$ for production capacity, e.g. the second time that the firm must decide if investing in $K_S$ to produce the drug is profitable. By assuming that the drug is worth investing in, the pharmaceutical firm invests in $K_S$, and the expected cash inflow from marketing the product or selling the R&D patent rights is $F$. This option can be obtained by applying the formula in Eq. (12). It can be expressed as follows:

$$
\tilde{C}_0(F, K_Y, K_S) = Fe^{\tilde{z}t_2}N_2\left(x', y', \rho\right) - K_Y e^{-\tilde{z}t_2}N_2\left(x' - \sigma_Y \sqrt{t_1}, y' - \sigma_Y \sqrt{t_2}, \rho\right)
$$

$$
- K_S e^{-\tilde{z}t_1}N\left(x' - \sigma_Y \sqrt{t_1}\right).
$$

The total value of this R&D project is equal to $\tilde{C}_0(F, K_Y, K_S) - I$.
For instance, a pharmaceutical company wanting to develop a curative drug for a common disease with a strong market potential initially invests $US 25 million in searching for active substances, as shown in $t = 0$ in Fig. 2. Following completion of the first stage in which several active substances are identified that are suitable for making this drug after three years, the company has decided to invest $US 90 million to determine whether the new drug would harm humans. If all 20 tests are completed at year 10 of the project, the company must determine whether to invest an additional $US 500 million in producing and marketing this new drug. As this R&D project is newly innovated, no market retail price for the drug product is available. All they can do is to find a twin project, which is tradable in the market. Assume that the expected cash inflow from marketing the twin product or selling the R&D patent rights for the twin project is $US 800 million and the risk-free rate on a ten-year government bond is 4%. Additionally, using the formula of our model in Eq. (12), allows us to obtain a “good-deal” bound for the compound option of this R&D project (258.099, 258.301) with respect to the value of 258.2 from Geske’s model. Further subtracting the cost of the initial investment, e.g. $US 25 million, which are sunk costs and not part of the option value, yields a total project value of (233.099, 233.301). Moreover, a situation in which the NPV criterion with a 14% weighted average cost of capital is used yields a value of $-4.82$US million.
Table 2. Values of good-deal bounds for R&D project in comparison with Geske’s model and NPV criterion.

<table>
<thead>
<tr>
<th>NPV</th>
<th>Upper Bound</th>
<th>Geske’s Value</th>
<th>Lower Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.82</td>
<td>233.301</td>
<td>233.200</td>
<td>233.099</td>
</tr>
</tbody>
</table>

Parameters Setting:

\[ t_0 = 0, \quad t_1 = 3, \quad t_2 = 10, \quad Y = 800, \quad \rho = 0.9, \quad r = 0.0039, \quad \sigma_a = 0.25, \quad \sigma_y = 0.25, \quad K_r = 90, \]
\[ K_s = 500, \quad \mu_s = 0.2, \quad \mu_f = 0.2. \]

These above values have many implications. First, real option methods are superior to NPV methods in evaluating R&D projects. Second, when markets fail, no observable price for R&D projects is available; a formula such as the one in this work must be established to identify the price bounds of the R&D project. Table 2 summarizes further details of the numerical analysis and parameter settings.

4. Hedging Strategies for Real Options

Real options literature has focused mainly on explaining how to evaluate real options and, to a lesser extent, hedging strategies. An R&D project includes two components: The static present value and the real option linked to the project. Although real options are another important component of firm value, feasible strategies to hedge against price moves have seldom been developed. For instance, due to deteriorating market conditions, some future stages of investment become less attractive, and a delay in investment lowers the value of real options. The entry of a competitor can also lower the value of real options. If the market is complete, the hedging strategy for a compound option can be expressed as follows:

\[
\frac{\partial CC(U_0, K_U, K_S)}{\partial S} = \frac{\partial CC(U_0, K_U, K_S)}{\partial U} = \frac{\partial x}{\partial U} = \frac{\partial y}{\partial U} = N_2(x, y, \rho) N(y),
\]

where

\[ x = \frac{\ln \left( \frac{U_0}{K_U} \right) + \left( r + \frac{1}{2} \sigma_U^2 \right) t_1}{\sigma_U \sqrt{t_1}}, \quad y = \frac{\ln \left( \frac{U_0}{K_U} \right) + \left( r + \frac{1}{2} \sigma_U^2 \right) t_2}{\sigma_U \sqrt{t_2}}. \]

By taking the partial derivative to Eq. (12) directly, the “approximate” delta hedge of a compound option under the condition of market incompleteness can be expressed as follows:
\[ \frac{\partial \tilde{C}C(Y_0, K_r, K_s)}{\partial S} = \frac{\partial \tilde{C}(y_t, K_r, K_s)}{\partial y} = N(y_2) \left( \bar{y}, \rho \right), \]  

where

\[ \bar{x} = \frac{\ln \left( \frac{Y_0}{K_r} \right) + \left( r + \xi + \frac{1}{2} \sigma^2 \right)t_1}{\sigma_r \sqrt{t_1}}, \quad \bar{y} = \frac{\ln \left( \frac{Y_0}{K_y} \right) + \left( r + \xi + \frac{1}{2} \sigma^2 \right)t_2}{\sigma_y \sqrt{t_2}}. \]

Eqs. (14) and (15) are quite similar. They differ in the term \( \xi \) in a normal distribution. This term seems to summarize all differences between market completeness and incompleteness.

Except for the compound option discussed in this work, Table 3 lists the hedging strategies for real options discussed in previous works.

Hedging strategies for real options whose underlying assets are not traded on the market are more difficult to implement than those in a complete market. However, neglecting the non-perfect correlation between the underlying and the surrogate security and simply using the standard hedge portfolio leads to sub-optimal hedging strategies.

5. Unresolved Difficulties

Although real options have been considered as a highly effective means of evaluating R&D projects and supplementing Discount Cash Flow (DCF) and Decision Tree Analysis (DTA) approaches, difficulties are encountered when applying options analysis to real world situations. According to Lint and Penning (1998) and Perlitz, et al. (1999), these problems can be summarized in four categories.

A. The exact exercise price and exercise date are uncertain

The possibility of exercise price for the R&D projects can be either a fixed one (McDonald and Siegel, 1986) or a stochastic one (Fischer, 1978; Lee and Paxson, 2001). The exercise price of an R&D project is normally unknown in advance. Modeling the investment cost of a certain R&D project as a stochastic process appears to be a more accurate means. The exercise date for the R&D projects seems to be uncertain as well. This date is normally influenced by many exogenous factors, including competition from rivals and first-mover advantages.

B. When market is incompleteness and risk neutral valuation is broken down

The risk neutral approach adopted in financial options appears to break down when real options evaluate R&D projects. This is owing to that adopting option valuation methods to value real assets seems to be incapable of forming a replicating portfolio as most real assets are normally non-traded or thinly traded. This work has focused on this topic. The method of obtaining a tradable twin asset as a proxy for non-tradable underlying assets is a widely used solution.

C. Estimating the volatility of the underlying asset is extremely difficult

Volatility is an index that evaluates the risk of the financial assets. However, for real options underlying real assets that are normally non-traded, estimating their volatilities is extremely difficult. Determining the volatility of an R&D project is problematic. Lint and Pennings (1998) indicated that since R&D projects are perceived as new, no historical
volatility data is available. However, by adopting a twin asset, the volatility may be derived from the duplicated portfolio. For instance, previous data from the volatility of similarly completed R&D projects can be applied to forecast the volatility of a new R&D project.

D. Real options are not necessarily exclusive options

Unlike financial options that are exclusive, real options can be either “exclusive” or “non-exclusive.” The value of an “exclusive” real option depends on the behavior of competitors. For instance, many firms can compete for a patent of a new R&D. When one of the firms completes the project, this can reduce the value of other firms. This characteristic of the projects is usually captured by the model with the Poisson jump process (Brach and Paxson 2001).

<table>
<thead>
<tr>
<th>Option Features</th>
<th>Studies</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Uncertain time horizons</td>
<td>Karoui and Martellini (2001)</td>
<td>Karoui and Martellini (2001) developed a feasible hedging strategy for a random cash flow to be received at a random date. Contingent claims analysis is performed by assuming that the investment opportunity has a known or infinite time horizon.</td>
</tr>
<tr>
<td>Transaction costs</td>
<td>Kabanov and Safarian (1997), Hoggard et al. (1994), Whalley and Wilmott (1997), Tompkins (2002)</td>
<td>Kabanov and Safarian identified the need for a highly constrained relationship between transaction costs and the rehedging frequency in the limit of the latter going to infinity. Hoggard et al. described the price of options as the solution to an integro-partial differential equation, as well as provided the limit hedging strategy for infinitesimal time periods between rehedgings.</td>
</tr>
<tr>
<td>Interdependence</td>
<td>Balmann and Musshoff (2001), Chidambaran (2000)</td>
<td>They proposed a genetic algorithm and a heuristic optimization scheme to value and hedge real options.</td>
</tr>
<tr>
<td>Stochastic volatility</td>
<td>Tompkins (2002)</td>
<td>Tompkins (2002) provided further insight into hedging problems associated with exotic options with stochastic volatility, as well as simulated these problems in a variety of contexts.</td>
</tr>
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6. Conclusions

Utilizing the real options pricing model to price R&D projects is increasingly more popular than adopting conventional methods, such as DCF and DTA. Considered as innovative, R&D projects are normally non-traded assets. Although this feature of R&D
projects has been mentioned previously, this difficulty has seldom been addressed by developing a feasible method. This work presents a novel compound option model that is constructed in the incomplete market to comply with the non-tradable feature of R&D projects. A situation is considered in which there is a “twin” or “surrogate” traded asset whose price process is highly correlated with that of non-traded underlying asset. An illustrative example of the cases shows two brands of drugs. One is to be developed shortly, and there is no liquid market for it; meanwhile, the other is a mature one, and there is a liquid market is available for it and allows for nearly frictionless trading. Although these two drugs have the same functionality and efficacy, the newly developed one is more effective. The observable market price of the mature drug allows us to price the value of the drug which is newly developed. The proposed model has two advantages. First, Geske’s model is incorporated to understand multistage characteristic of R&D projects. Second, a model is established in an incomplete market to understand another characteristic of R&D projects as non-tradable assets. Although one of the limitations of previous models that treat the underlying non-traded asset as a traded one is revoked, the option pricing model used in previous works and the proposed model still contains many drawbacks. As for the limitations of this study, such as the cases discussed in Section 5 (uncertain of real option exercise day, volatility estimation of R&D projects, the value of exclusive real option depends on the behavior of competitors), they must still be resolved. Moreover, the proposed model based on Geske’s compound option model is only appropriate for evaluating two-stage projects. The proposed model can also be extended to the projects with n-stages by applying the method of Cortazar and Casassus (2000), yet requires a more complex numerical method and added complexity. Future work should address the above concerns.

References


