Measurement Model of Risk Compensation: Evidences from Asian Emerging Markets

Shu-Hsien Chen\textsuperscript{a}, Ming-Shann Tsai\textsuperscript{b}, Phil Y. Yang \textsuperscript{c,}\textsuperscript{*}

\textsuperscript{a}Department of International Business, Central Taiwan University of Science and Technology, Taiwan
\textsuperscript{b}Department of Banking and Finance, National Chi Nan University, Taiwan
\textsuperscript{c}Graduate Institute of Business Administration, National Taichung University, Taiwan

Accepted 10 February 2009

Abstract

This study investigates the measurement of investment weight adjustment on jump risk of five Asian emerging markets. Considering the risk of rare event, we develop a stochastic jump-diffusion model for dynamic asset allocation in an international diversified portfolio. This paper contributes to finding the optimal weight on jump risks that differs from prior research by calculating the jump size and event arrival frequencies. A covariance of international assets investment is obtained by simulating the price process and constructing multiple-dimension jump diffusion on weight adjustment. We also show how to diversify the jump risks by using international assets portfolios in Asian emerging markets.

Keywords: Rare events, dynamic asset allocation, international diversified portfolio, jump risk

1. Introduction

Traditional finance analysts think that investors with the concept of global diversification can hedge some political or economic risks by diversifying equities across many countries. Several studies have identified political risk or economic risk as one of the most important issues for managers when dealing with foreign investments (Lensink et al., 2000; Micallef, 1981; Mortanges and Allers, 1996). Hence, wealth compensation has become a relative parameter in portfolio management. Perotti and Oijen (2001) present empirical evidence in emerging economies, and suggest that progress in privatization is indeed correlated with improvements in perceived systematic risk. Their analysis further shows changes in event risks caused by the general trend of a strong effect on local stock market development and the excess returns in emerging economies.

Moreover, Kim and Mei (2001) adopt a component jump volatility filter to investigate the rare event risk on the stock returns of Hong Kong. Their results show that event risks have a significant impact on their market volatility and returns. The discussion on how to benefit from investment diversification has become an important issue. In their recent work, Liu et al. (2003) have indentified that rare events often trigger unexpected changes in equity price and volatility. They provide analytical solutions to the optimal portfolio problem and point out implications of jumps on price in investment strategies. Thereafter, Das and Uppal (2004) distinguish the literature on portfolio choice from that on event risks.

* Corresponding author. E-mail: ysyang@ntcu.edu.tw
The emergence of developing countries in East Asia is a main characteristic of global economic change in the last 30 years. The reason is that Asian emerging markets have shown strong economic growth patterns among the world’s economic powers. Even though economic growth in western developed countries has declined since the 1990s, many countries in East Asia have maintained high economic development. Many companies in developed countries have recently expanded operations into emerging markets; investors are now willing to look for investment opportunities in Asia. Since rare events in market cause the price of an asset to jump, attention has been given to the growth prospects of the emerging Asian markets such as China, India, South Korea, Thailand and Taiwan. Even though these markets have quite different economic histories, all of them have shown high growth rates and have the potential to continue these trends.

The centre of global economic prosperity has shifted to the East. Specifically in 2004, the total GDP of China, Japan and South Korea exceeded by U.S. $7 trillion, which accounted for 1/5 of the world economy. During the same period, the amount of trade in the Asian-Pacific region soared to $15 billion, of which China and India accounted for $3 billion. Despite of the Asian financial crisis in 1997, South Korea rose to become the 9th largest economy, while Taiwan remained the 23rd largest economy in the world. Thailand’s recovery from the Asian financial crisis in 1997 relied on exports, largely from external demands, namely from the U.S. Since then, Thailand’s economic policy combines domestic stimulus with the country’s traditional promotion of open markets and foreign investment. There is a tendency to invest in Asian emerging economies. Investor may be interested in optimal portfolio weights among Asian countries. This study selects those five emerging Asian markets’ stock indexes of five emerging Asian markets (as mentioned above namely, China, India, South Korea, Taiwan, and Thailand) into the portfolio. Specifically, the monthly returns on stock indexes of these five emerging Asian countries are calculated.

This paper aims to establish an efficient and effective method for evaluating the weight changes of investment caused by jump risk in the emerging markets. Jump risk of rare events poses strong effects on excess returns in the emerging stock market. The method of dynamic asset allocation is widely employed to determine optimal portfolio weights (i.e. Merton, 1990). Chen et al. (2006) discuss how the conditional moment’s link influences the effects of outcome risk on portfolio decisions. They solve the expectation term in the optimal portfolio model. However, there are still many problems that have to be solved using this method, particularly for event-driven uncertainty, evade intensities, and jump sizes. Therefore, the observations in a portfolio are difficult to measure, assess and adjust.

Some researchers utilize continuous-time generalization assuming the exogenous equity return to follow a jump-diffusion process (Bekaert et al., 2002; Merton, 1990; Payne, 2003). The general equilibrium treatment of jumps, for example, Kou (2002) uses the Black–Scholes option-pricing framework to model the return of assets. Aït-Sahalia (2004) uses a characterization of the transition function of diffusion and Wu (2003) uses short-dated options. In this study, we investigate the effect of hedging the jump risk on rare events by adjusting the portfolio weight.

Moreover, prior studies on jump diffusion portfolio model expect to discover the solution to obtaining the optimal portfolio weight (e.g. Das and Uppal, 2004; Liu et al., 2003; Runggaldier, 2002; Wu, 2003). This study develops an improvement to determine the correlation of variance between each jump of two assets, which is more realistic for calibrating the empirical result of financial asset prices used in portfolio choice (e.g. Aït-Sahalia, 2004; Bentzen and Sellin, 2003). Alder and Dumas (1983) point out that the optimal portfolio combines the weights of two components in the standard mean variance model. The first component is the portfolio of a logarithmic investor. The formula indicates that its composition is independent of the behavior of commodity prices. Therefore, commodities
prices are not included in the objective function and have no influence on the decisions. The second component portfolio is the portfolio of an investor with zero risk tolerance. It is therefore, for any given investor, his global minimum variance portfolio in real terms. In our model the return volatility is driving asset allocation.

We arrange this paper in other sections as follows. In Section 2, the basic model for the solution has been found for the optimal portfolio weight. Section 3 calibrates an empirical evidence for the emerging markets. We analyze the result of investment weight adjustment in section 4, and in Section 5, we arrive at a conclusion.

2. Optimal portfolio model

This study sets a model to find the optimal investment weight in five emerging Asian markets. There are two kinds of assets including a riskless asset bond (B) and risky asset (S). The return of the bond follows the process

\[ \frac{dB}{B} = rdt, \]

where \( B \) represents a riskless asset bond with return \( r \).

There are five risky assets whose prices \( S = [S_1, S_2, ..., S_5]' \) are subject to event-related jumps. In continuous time finance, a popular way to generate discontinuity is to apply the compound Poisson Jump Model. Therefore, the returns of five risky assets, \( \frac{dS_i}{S_i}, i = 1, ..., 5 \), follow a jump-diffusion process by using the next formula:

\[ \frac{dS}{S} = (\mu - r \theta)dt + \sqrt{\nu}dZ + Xdq \]

where \( \frac{dS}{S} = [\frac{dS_1}{S_1}, \frac{dS_2}{S_2}, ..., \frac{dS_5}{S_5}]' \) is the returns matrix of the five risky assets, \( \mu = [\mu_1, \mu_2, ..., \mu_5]' \) is the vector of the mean return of five risky assets and \( r = [r, ..., r]'_r \) is the vector of riskless interest rate. \( \theta = [\theta_1, \theta_2, ..., \theta_5]' \) is the mean of jumps size of the five risky assets. The term \( \lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_5 \\ 0 & \cdots & \lambda_5 \end{bmatrix} \) captures the mean percentage jump in the asset price conditional on jump occurrence and different jump arrivals. The probability from a Poisson process with stochastic arrival intensity \( \lambda_i \) is obtained from the occurrence probability \( \Pr(dq_i = 1) = \lambda_i dt \) and \( E[dq_i] = \lambda_i, i = 1, ..., 5 \) which we shall discuss in Section 3.

The matrix of the jumps arrival index is \( Q = \begin{bmatrix} dq_1 & 0 \\ \cdots & \cdots \\ 0 & dq_5 \end{bmatrix} \). The event-related jumps size of five risky assets is denoted by vector \( X = [X_1, X_2, ..., X_5]' \).

Therefore,
V = \sigma \rho \sigma

= \begin{bmatrix}
\sigma_1 & 0 & \cdots & 0 \\
0 & \sigma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_5
\end{bmatrix}
\begin{bmatrix}
1 & \rho_{1,2} & \cdots & \rho_{1,5} \\
\rho_{2,1} & 1 & \cdots & \rho_{2,5} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{5,1} & \rho_{5,2} & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\sigma_1 & 0 & \cdots & 0 \\
0 & \sigma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_5
\end{bmatrix}
stands

for the 5×5 variance-covariance matrix of 5 risky assets which standard deviation is denoted by \( \sigma \). The correlation between asset \( i \) and asset \( j \) is indicated by \( \rho_{ij} \) and the vector of standard Brownian motion of five risky assets is defined as
\[ dZ = [dZ_1, dZ_2, \ldots, dZ_5] \sim N(0, Idt) \).

Subsequently, this study turns to a portfolio decision by applying the Taylor expansion to the Euler equation and approximating the optimal portfolio choice. Maximizing the expected utility of the terminal wealth \( W_t \), i.e. \( \max_{\phi} E[U(W_t)] \), and the return of wealth process satisfy the self-financing condition. This study arrives at the dynamic wealth process equation:

\[
\frac{dW}{W} = \phi_0 \frac{dB}{B} + \phi \Phi dS,
\]

where \( \phi_0 \) represents the weight on risk-free asset, \( \Phi = [\phi_1, \phi_2, \ldots, \phi_5]^T \) is a vector of a portfolio which indicates the investor’s portfolio choice among the available investment opportunities.

In searching for the optimal portfolio strategy, this study adopts the standard stochastic control approach and assumes that the jump arrival probability is constant. The principle of optimal stochastic control leads to the following Hamilton-Jacobi-Bellman (HJB) equation by Merton (1990, Chapter 5) for the indirect utility function \( J \):

\[
\max_{\phi} \left\{ \frac{\partial J}{\partial t} + \frac{\partial J}{\partial W} E(dW) + \frac{1}{2} \frac{\partial^2 J}{\partial W^2} Var(dW) + E[J(W(1+\Phi^TQX),t) - J(W,t)] = 0. \right. \tag{4}
\]

This study searches for the optimal portfolio strategy \( \Phi^* \) by first conjecturing that the indirect utility function is of the form

\[
J(W,t) = \frac{1}{1-\gamma} W^{1-\gamma} \exp(A(t)),
\]

where risk aversion coefficient \( \gamma > 0 \), \( \gamma \neq 1 \) and \( A(t) \) is a function of time but not of the state variables \( W \). Given this function form, this study takes derivatives of \( J(W,t) \) with respect to its arguments, \( J_W = W^{-\gamma} e^A, J_{WW} = -\gamma W^{-\gamma-1} e^A, \) substituting it into the HJB equation in Equation (4). Subsequently we arrive at

\[
\max_{\phi} \left\{ J_t + W^{-\gamma} e^A \left[r + \Phi^T(\mu - r - \lambda \theta)\right] dt + \frac{1}{2} (-\gamma W^{-\gamma+1}) e^A \Phi^T V \Phi dt \right.
\]

\[
\left. + E\left[ \frac{1}{1-\gamma} (W(1+\Phi^TQX))^{1-\gamma} e^A - \frac{1}{1-\gamma} W^{1-\gamma} e^A \right] \right\} = 0. \tag{5}
\]

We differentiate Equation (5) with respect to the portfolio weight of the risky asset, \( \Phi \), and divide by \( W^{-\gamma} e^A dt \) to obtain the following first-order condition:

\[
(\mu - r - \lambda \theta) - \gamma \Phi^* V^* + E[(1+\Phi^*QX)^{-\gamma} QX] = 0. \tag{6}
\]

From Equation (6) the optimal portfolio weight can be expressed by
\[ \Phi^* = V^{-1} \left[ \frac{\mu - r - \lambda \theta}{\gamma} + E[(1 + \Phi''\Omega X)^{-\gamma} \Omega X] \right] \] (7)

for each \( \phi_i^* \in \Phi^* \).

The traditional solution of Equation (7) for the optimal portfolio weight is inadequate because there is an unknown expectation value as an implicit function (Das and Uppal, 2004; Liu et al., 2003; and Wu, 2003). In general solving for the unclosed form after setting the jump size distribution, traditional research usually applies a numerical method (i.e. simulation) to find the value of optimal portfolio weight, \( \Phi^* \).

Here, in the solution for optimal portfolio weight in Equation (7), we place little attention to the joint distribution to the jump size \( X \). For the expectation \( E[(1 + \Phi''\Omega X)^{-\gamma} \Omega X] \), if each element in \( \Phi'' \Omega X \) is small (i.e. \( \phi_i^* \), \( X \) < 1), then it follows that (see the detail in Appendix A):

\[ E[(1 + \Phi''\Omega X)^{-\gamma} \Omega X] \approx E[e^{-\gamma \Phi'' \Omega X} \cdot \Omega X] \]

The 5×5 variance-covariance matrix of jumps size of 5 risky assets can be expressed by

\[ \Omega = \begin{bmatrix} \delta_{1} & 0 & \cdots & 0 & \rho_{1,2} & \cdots & \rho_{1,5} \\ 0 & \delta_{2} & \cdots & 0 & \rho_{2,3} & \cdots & \rho_{2,5} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_{i} & \rho_{3,1} & \cdots & 1 \end{bmatrix} \]

represents for the standard deviation of jump with row and column index \( i \). Substitution of Equation (8) in (7), gives the following expression for the optimal portfolio weight

\[ \Phi^* = V^{-1} \left[ \frac{\mu - r - \lambda \theta}{\gamma} + \frac{(\lambda \theta - \gamma \Phi'' \Omega \Phi^*)}{\lambda^2} \right] \exp[-\gamma \Phi'' \Omega \Phi^*] \] (9)

Thus, this study finds a closed form solution \( \Phi^* = f(\mu, \nu, \lambda, \theta, \Omega, r) \) for the optimal portfolio weight of 5 risky assets as shown in Equation (9) which improves the earlier work in Equation (7).

With respect to verify the jumps that exists in the emerging markets, this study observes the estimation of weight \( \Phi^* \) by the endogenous variables \( \mu, \nu, \lambda, \theta, \Omega \). A popular method for jump risk parameters estimation is developed by Press (1967) and Beckers (1981), who employ a version of the method of moments known as the cumulant matching method. In their specification, the mean jump amplitudes and the remaining parameters, \( \mu_i, \theta_i, \sigma_i, \delta_i \) (\( i = 1, 2, \ldots, 5 \)) and \( \lambda \) are endogenous. Therefore,

\[ \ln \frac{S(T)}{S(t)} \sim \sum_{i=1}^{\infty} \frac{e^{-\lambda \tau} \left( \lambda \tau \right)^i}{i!} N \left( \mu \tau - i \theta, \sigma^2 \tau + i \delta^2 \right), \]

where \( \tau = T - t \), the log return distribution is described as a Poisson mixture of normal distribution. Using the relationship between cumulants and moments (Kendall and Stuart, 1963), it can be proved that the log return distribution is leptokurtic and therefore might be better to describe the actual stock price return than the pure lognormal model. The formulae for the
solution of parameters, $\mu_i, \theta_i, \sigma_i, \delta_i$ ($i = 1, 2, \ldots, 5$) and $\lambda$ in Press (1967) and Beckers (1981) are shown in Appendix B.

However, the formulae for estimating the parameters demand unreasonable assumptions of mean return zero or jump mean return zero, either in Press (1967) or Beckers (1981). The alternative method for estimation in jump risk portfolio model is the Maximum Likelihood Estimates (MLE). This study follows the process in Bentzen and Sellin (2003) that investigates jump risk in the market portfolio with different types of log-likelihood function. If there are no jumps in each country, the log-likelihood function can be written as:

$$
\ln L_{no} = -\frac{T}{2} \ln(2\pi \sigma^2 h) - \sum_{i=1}^{T} \frac{(\ln s_i - \mu h)^2}{2\sigma^2 h},
$$

where $T$ is the number of observations, $h$ is the increment of time between observations and $s_i \equiv \frac{S_i}{S_{i-1}}$. Otherwise, if there are jumps, the log-likelihood function in each country can be written as follows:

$$
\ln L_{jp} = -\frac{T}{2} \ln(2\pi) - T \lambda h + \sum_{i=1}^{T} \ln\left(\sum_{j=0}^{\infty} \frac{(\lambda h)^j}{j!} \frac{1}{\sqrt{\sigma^2 + \delta^2 j}} \exp\left(-\frac{(\ln s_i - \mu h - \theta j)^2}{2(\sigma^2 + \delta^2 j)}\right)\right).
$$

The likelihood ratio statistic, $LR = -2(\ln L_{no} - \ln L_{jp})$, is distributed asymptotically $\chi^2$ with the degree of freedom 3. This paper follows Bentzen and Sellin (2003) which selects the value of $j$ from zero to ten. The estimates are given for the combined diffusion and jump process. A simple likely ratio test of the jump parameter indicates that for some of the daily sample periods a statistically significant jump component exists. This is confirmed by the likelihood ratio test.

3. Benchmark portfolio and risk compensation

Assume that returns are described by only the pure-diffusion process in Equation (9), the set of benchmark portfolio weights is

$$
\Phi^* = \gamma^{-1} V^{-1}(\mu - r)
$$

where $\Phi^*$ is the benchmark portfolio.

The compensation of the rare event jump can be obtained by the expectation of investing the same wealth in two cases: the first case of the optimal portfolio weights on benchmark $\Phi^*$ without considering jump risks and the second case of the optimal portfolio weights $\Phi^*$ with considering jump risks in the future.

Now, the calculation of the risk compensation on wealth by following the framework is designed by Das and Uppal (2004). Because of ignoring systemic jumps the notation of the compensating wealth $\Delta \xi^*$ is computed as follows:

$$
J((1 + \Delta \xi^*)W_t,t; \Phi^*) = J(W_t,t; \Phi^*)
$$

where $J$ is the conjecture value function. According to Equation (5) the maximize wealth satisfied a function of the optimal portfolio weight, $\Phi^*$, as the following condition:

$$
J\mu W[r + \Phi^*(\mu - r - \lambda \mu x)]dt + \frac{1}{2} J_{\Phi^*\Phi^*}W^2 \Phi^* V \Phi^* dt
$$
Suppose that \( \Lambda(t; \Phi) \) is a function relative to time \( t \) and portfolio weight \( \Phi \), except the relative to wealth. It is described by

\[
J(W, t) = \frac{1}{1 - \gamma} W(\Phi)^{1 - \gamma} e^{A(t)} \equiv \frac{1}{1 - \gamma} W^{1 - \gamma} e^{A(t; \Phi)}.
\]

Therefore the additional benefit gains (i.e. \( \Delta \xi \)) by considering the jump risk that will be the risk compensation on wealth from the adjustment of optimal portfolio weights. It can be expressed by the following equation:

\[
E[e^{\Lambda(t; \Phi)} \frac{1}{1 - \gamma} ((1 + \Delta \xi)W(\Phi^*))^{1 - \gamma}] = E[e^{\Lambda(t; \Phi)} \frac{1}{1 - \gamma} (W_0(\Phi^*))^{1 - \gamma}]
\]

(12)

The benefit obtained by wealth arrangement can be simplified as follows:

\[
\Delta \xi = e^{\Lambda(t; \Phi^*) - \Lambda_0(t; \Phi^*)} - 1
\]

(13)

To identify the value of maximizing the expectation term, \( E[\frac{W(\Phi^*)}{W_0(\Phi^*)}] \). Then, substitute

\[
J_W = W^{-\gamma} e^{\Lambda(t; \Phi)}, \quad J_{WW} = -\gamma W^{-\gamma - 1} e^{\Lambda(t; \Phi)} \quad \text{and} \quad J(W, t) = \frac{1}{1 - \gamma} W^{1 - \gamma} e^{A(t; \Phi)} \quad \text{in Equation (12)}
\]

and is divided by the common factor \( J \) in both sides, we obtain

\[
\frac{\partial \Lambda(t; \Phi)}{\partial t} \equiv -\kappa
\]

where

\[
\kappa \equiv (1 - \gamma)[r + \Phi^*(\mu - r - \lambda \Phi_x)] - \frac{1}{2}(\gamma)(1 - \gamma)\Phi^* \mu \Phi^* + \lambda (\mu_x - \gamma \Phi^* \Phi_x) \cdot \exp(\gamma \Phi^* \mu + \frac{1}{2}\gamma^2 \Phi^* \Phi_x \Phi^*)
\]

(14)

Therefore, \( \Lambda(t; \Phi) = \int_0^t -\kappa d\tau = -\kappa(T - t) \). On the contrary, we replace the benchmark portfolio weights \( \Phi^* \) in Equation (13), it results

\[
\frac{\partial \Lambda(t; \Phi)}{\partial t} \equiv -\bar{\kappa}
\]

where

\[
\bar{\kappa} \equiv (1 - \gamma)[r + \Phi^*(\mu - r)] - \frac{1}{2}(\gamma)(1 - \gamma)\Phi^* \mu \Phi^*
\]

(15)

\( \kappa \) and \( \bar{\kappa} \) are the time impact factors on wealth with jump and without jump, respectively. By substituting \( \kappa \) and \( \bar{\kappa} \) in Equation (11) of the value function to find the risk compensation on wealth, the wealth reward is obtained.
$$\Delta \xi = E \left( \frac{e^{\kappa}}{e^{\bar{\kappa}}} \right)^{1/(1-\gamma)} - 1 = e^{(\kappa - \bar{\kappa})(T-t)} - 1.$$  

4. Empirical results

For simplicity purpose, this study investigates investment in risky and riskless assets without any transaction costs and restrictions. The monthly data were retrieved from Morgan Stanley Capital International Inc. (MSCI) database using the sample period from January 1993 to November 2006. The data of Figure 1(a) to Figure 1(e) were extracted from MSCI market capitalization and Central Banks on the World Wide Web with respect to the estimated measurement of stock price jump trend versus foreign investment from 1993 to 2006. The development trends show the stock prices increase (or decrease) abruptly. And the largest change is with the foreign investment measure showing sharp increases (or decreases) in India, Taiwan and Thailand. Although China and Korea have shown mitigate increases, this evidence is consistent and the capital changed after equity market liberalizations.

![Figure 1(a). The correlation of stock index and foreign investment of China.](image)

![Figure 1(b). The correlation of stock index and foreign investment of India.](image)
Correlation of stock index and foreign investment in Korea

![Graph showing the correlation of stock index and foreign investment in Korea.](image)

Figure 1(c). The correlation of stock index and foreign investment of Korea.

Correlation of stock index and foreign investment in Taiwan

![Graph showing the correlation of stock index and foreign investment in Taiwan.](image)

Figure 1(d). The correlation of stock index and foreign investment of Taiwan.

Correlation of stock index and foreign investment in Thailand

![Graph showing the correlation of stock index and foreign investment in Thailand.](image)

Figure 1(e). The correlation of stock index and foreign investment of Thailand.
Table 1 summarizes the statistical properties of the stock returns for the five emerging Asian markets and the world market. As can be seen, the five emerging Asian markets all have positive mean returns during that period. In contrast to a normal distribution, we have obtained an empirical distribution with a negative skewness in the world index return and Indian stock index return. The kurtosis measures indicate that the return distribution peaked more (or fat tails) than the normal (i.e. 5.7889 in China, 4.6911 in Thailand, 6.0788 in Taiwan and 9.3746 in South Korea. These protruding curves show a significant event jump of stock returns.

Simple bivariate correlations on stock indexes between the five emerging Asian markets are presented in Table 2. In general, investors select the low correlation assets as their portfolio. We find that there are positive and low correlations between each of the two countries in the five emerging Asian markets. The highest mean returns correlation of 0.5694 is that between Thailand and South Korea, followed by that of 0.5086 between China and Taiwan.

Table 1. Statistics of stock return for five emerging Asian markets.

<table>
<thead>
<tr>
<th>Market Indices</th>
<th>Mean $\hat{\mu}$</th>
<th>St. Dev. $\hat{\sigma}$</th>
<th>Max</th>
<th>Min</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Bera-Jarque</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>0.0231</td>
<td>1.3147</td>
<td>0.4650</td>
<td>-0.2767</td>
<td>0.7928</td>
<td>5.7531</td>
<td>246.32*</td>
</tr>
<tr>
<td>India</td>
<td>0.1403</td>
<td>0.9915</td>
<td>0.2200</td>
<td>-0.1774</td>
<td>-0.0136</td>
<td>2.5797</td>
<td>46.03</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.1597</td>
<td>1.4530</td>
<td>0.7059</td>
<td>-0.3126</td>
<td>1.2629</td>
<td>9.3175</td>
<td>644.60**</td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.0986</td>
<td>1.1490</td>
<td>0.4644</td>
<td>-0.2187</td>
<td>1.0155</td>
<td>6.0433</td>
<td>281.14*</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.0474</td>
<td>1.4977</td>
<td>0.4318</td>
<td>-0.3405</td>
<td>0.4279</td>
<td>4.7172</td>
<td>158.98*</td>
</tr>
</tbody>
</table>

Notes: 1. Sample period for MSCI index return monthly data is retrieved from January 1993 to December 2006. 2. The stock indices are retrieved from the world index in MSCI. 3. * stands for 0.5 and ** stands for 0.9 of the probability significance levels, respectively.

Table 2. Bivariate correlations between five emerging Asian markets.

<table>
<thead>
<tr>
<th>Indices</th>
<th>China</th>
<th>India</th>
<th>South Korea</th>
<th>Taiwan</th>
<th>Thailand</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>1.0000</td>
<td>0.2794</td>
<td>0.2412</td>
<td>0.5086</td>
<td>0.4897</td>
</tr>
<tr>
<td>India</td>
<td>1.0000</td>
<td>0.2413</td>
<td>0.3233</td>
<td>0.3233</td>
<td>0.22597</td>
</tr>
<tr>
<td>South Korea</td>
<td>1.0000</td>
<td>0.3620</td>
<td>0.56941</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taiwan</td>
<td>1.0000</td>
<td>0.47454</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Simple bivariate correlations between five emerging Asian markets are presented.

Thus, this study uses the log-likelihood function for estimating the mixed Poisson jump-diffusion model and the simple Wiener process model of monthly returns to the stock market indices of the five emerging Asian markets. The likelihood ratio test is employed to test the hypothesis of whether jump risk exists. We have the jumps by calibrating the real world data into the models. By investigating the monthly data under the condition of event instability, this study presents the parameter estimates for the jump-diffusion model of emerging markets as estimated by the cumulant matching method shown in Table 3. As seen in Table 1 and the bottom panels of Table 3, the results obtained using different measurements in estimating the
mean and standard deviation show slight differences. Cumulant matching method was found to yield more accurate figures.

Table 3. Estimates of monthly returns of the five emerging Asian stock markets by cumulant matching method.

<table>
<thead>
<tr>
<th>Indices with jump</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}$</th>
<th>$\hat{\lambda}$</th>
<th>$\hat{\theta}$</th>
<th>$\hat{\delta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>-0.0741</td>
<td>0.4028</td>
<td>0.2415</td>
<td>0.1047</td>
<td>0.0034</td>
</tr>
<tr>
<td></td>
<td>(0.0957)</td>
<td>(0.1009)</td>
<td>(0.3281)</td>
<td>(0.0132)</td>
<td>(0.0315)</td>
</tr>
<tr>
<td>India</td>
<td>0.1403</td>
<td>0.0230</td>
<td>0.0000</td>
<td>0.0814</td>
<td>0.0042</td>
</tr>
<tr>
<td></td>
<td>(0.1639)</td>
<td>(0.0000)</td>
<td>(7.1583**)</td>
<td>(0.0289)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>South Korea</td>
<td>0.0027</td>
<td>0.0521</td>
<td>3.096</td>
<td>0.0676</td>
<td>0.0312</td>
</tr>
<tr>
<td></td>
<td>(0.0986)</td>
<td>(0.0382)</td>
<td>(1.5104)</td>
<td>(0.0148)</td>
<td>(0.0155)</td>
</tr>
<tr>
<td>Taiwan</td>
<td>-0.0525</td>
<td>0.1906</td>
<td>0.7926</td>
<td>0.0712</td>
<td>0.0125</td>
</tr>
<tr>
<td></td>
<td>(0.1208)</td>
<td>(0.5493)</td>
<td>(2.7398*)</td>
<td>(0.0167)</td>
<td>(0.0604)</td>
</tr>
<tr>
<td>Thailand</td>
<td>-0.1541</td>
<td>0.3314</td>
<td>0.6079</td>
<td>0.1223</td>
<td>0.0025</td>
</tr>
<tr>
<td></td>
<td>(0.1474)</td>
<td>(0.1232)</td>
<td>(0.8049)</td>
<td>(0.0160)</td>
<td>(0.0273)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Indices without jump</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}$</th>
<th>$L. R. test$</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>0.0231</td>
<td>0.1431</td>
<td>21.8840*</td>
</tr>
<tr>
<td></td>
<td>(0.1089)</td>
<td>(0.0109)</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>0.14035</td>
<td>0.0814</td>
<td>0.2300**</td>
</tr>
<tr>
<td></td>
<td>(0.0765)</td>
<td>(0.0101)</td>
<td></td>
</tr>
<tr>
<td>South Korea</td>
<td>0.1597</td>
<td>0.1749</td>
<td>36.7240</td>
</tr>
<tr>
<td></td>
<td>(0.1247)</td>
<td>(0.0104)</td>
<td></td>
</tr>
<tr>
<td>Taiwan</td>
<td>0.0986</td>
<td>0.1094</td>
<td>21.7550*</td>
</tr>
<tr>
<td></td>
<td>(0.0994)</td>
<td>(0.0085)</td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>0.0474</td>
<td>0.1858</td>
<td>15.1340*</td>
</tr>
<tr>
<td></td>
<td>(0.1185)</td>
<td>(0.0153)</td>
<td></td>
</tr>
</tbody>
</table>

Note: $\hat{\mu}$ is the estimated mean of stock return; $\hat{\sigma}$ is the estimated standard deviation of stock return; and, a Poisson process with stochastic estimated arrival intensity $\hat{\lambda}$. A random percentage jump size with estimated mean $\hat{\theta}$, and $\hat{\delta}$ is the estimated standard deviation of random percentage jump size of stock return. Standard errors are reported in the parenthesis. * stands for 0.5 and ** stands for 0.9 of the probability significance levels, respectively.

Because investors will withdraw the capital out from the portfolio when the return of equity shows a downward jump, thus we adopt the upward jump size (i.e. $\hat{\theta} \geq 0$) for the portfolio. Table 3 shows that if we consider the return with pure diffusion the means for five countries are all positive (with no investor withdrawing the capital). On the other hand, when we consider the return with jump diffusion of equities, the means in China, Taiwan and Thailand turn to negative. The event jump arrival frequency in stock index return of India is the smallest among these five countries in stock index return. The final column gives the likelihood ratio test of the null hypothesis, that is, for these five Asian countries, the portfolio calculated by the pure diffusion model can diversify the risk better than the portfolio of the alternative hypothesis calculated by jump-diffusion model. Although the estimated values for
the probabilities of jumps are larger than the critical value \((p = 0.05)\), it is clear that the model with a jump component gives a much better fit to the data. The likelihood ratio tests are all significant compared with \(\chi^2\) with three degrees of freedom (in our case \(5 - 2 = 3\) d.f.) and the critical value \((p = 0.05) = 7.82\). Therefore, the results of the likelihood ratio test are significant and the null hypothesis is rejected on all these countries, except India. The results in Table 3 verify that pure diffusion model cannot diversify the risk better than jump diffusion model.

The economic implication for rejecting the null hypothesis means that jump risk cannot be diversified and an instantaneous CAPM will not hold. Moreover, if include the weekend and holiday returns, the observed jumps are accentuated. This is consistent with the reduction of the likelihood ratios. This model captures the idea of discrete changes by introducing a stochastic jump process. The market portfolio contains a jump component in Asian emerging markets (except India) although its magnitude is small. Obtained from the likelihood ratio test, we find that the effects on portfolio management by considering with and without jump risks are significant in South Korea, China Thailand and Taiwan.

The cumulant matching method gives the results when estimating the mean number of jumps. However, the method provides the means and standard deviations of return and jumps, respectively. We still need to find the correlation of variance \((\rho_{ij})\) with jumps and without jumps by the maximum likelihood estimates. In previous work (Liu et al., 2003; Runggaldier, 2002), the optimal portfolio weight cannot be found due to an unknown expected value in the solution of portfolio weight and the covariance of pure diffusion and covariances of jump diffusion unidentified. In this paper, we attempt to solve the expectation term by joint probability density function and the investigation of the covariance between each two from the five emerging Asian markets through the approach of 1000 times Monte Carlo Simulation with jump diffusion and pure diffusion. This process is available for calculating the optimal portfolio weight in Equations (9) and (10). Table 4 shows that the stock index return with considering jump of Taiwan has a negative covariance with those of India, South Korea and Thailand markets. The monthly return with considering jump of the Thailand stock market has a negative variance correlation with the rest four countries. However, if jump-diffusion is not considered, China stock return has merely a positive covariance correlation with South Korea.

Table 4. Estimates of variance correlations \((\rho_{ij})\) between two emerging Asian markets by Monte Carlo Simulation.

<table>
<thead>
<tr>
<th>Indices</th>
<th>China</th>
<th>India</th>
<th>South Korea</th>
<th>Taiwan</th>
<th>Thailand</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: with jump-diffusion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>1</td>
<td>0.00405</td>
<td>0.00049</td>
<td>0.00134</td>
<td>-0.00001</td>
</tr>
<tr>
<td>India</td>
<td>1</td>
<td>0.00078</td>
<td>-0.00004</td>
<td>-0.00470</td>
<td></td>
</tr>
<tr>
<td>South Korea</td>
<td>1</td>
<td>-0.00537</td>
<td>-0.00232</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taiwan</td>
<td>1</td>
<td>-0.00031</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel B: without jump-diffusion (pure diffusion)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>1</td>
<td>-0.00234</td>
<td>0.00254</td>
<td>-0.00089</td>
<td>-0.00050</td>
</tr>
<tr>
<td>India</td>
<td>1</td>
<td>-0.00131</td>
<td>-0.00003</td>
<td>-0.00428</td>
<td></td>
</tr>
<tr>
<td>South Korea</td>
<td>1</td>
<td>0.00416</td>
<td>0.00075</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taiwan</td>
<td>1</td>
<td>0.00141</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Moreover, we need to specify the riskfree rate and the agent’s relative risk aversion. We assume annual riskfree rate = 0.02 per year which is close to the average U.S. 1-month riskless interest rate for the U.S. investor and we set the base-case relative risk aversion, \( \gamma \), equal to 3.0. With these parameter values, we solve numerically the first-order conditions in Equation (10) to obtain the optimal portfolio weights, \( \Phi^* \), for an investor who accounts for systemic risk and the weights, \( \bar{\Phi}^* \), of the investor who ignores systemic jumps and assumes returns are given by a pure-diffusion process. We also report the composition of the portfolio consisting of only risky assets to these portfolio weights which can be obtained by dividing each individual weight by the total investment in risky assets. These weights are given by \( \frac{\phi}{\sum \phi} \) (or \( \Phi^*/(\Phi^* \cdot 1) \)) for the systemic-jump case and \( \frac{\phi}{\sum \phi} \) (or \( \bar{\Phi}^*/(\bar{\Phi}^* \cdot 1) \)) for the pure diffusion case.

This study verifies that investment weight adjustment to portfolio fraction in India has been reduced when the jump risk is underestimated. The portfolio weight on the South Korea index return has a 0.0101 (= 1.6632 - 1.6531) increased which is the only positive effect in considering jumps among the five countries in Table 5. By the way, suppose that the wealth is combined to one by weight on risk-free asset \( \phi_0 \) and the investor’s portfolio choice \( \Phi = [\phi_1, \phi_2, ..., \phi_5] \), i.e. \( \phi_0 + \Phi^* \cdot 1 = 1 \). In this case, portfolio weights with pure diffusion have the sum of 3.467. It means to short sale the riskless asset of 2.467 and to buy this portfolio, but when we consider about the jumps we only short sale the riskless asset of 1.918 for gaining more return.

We compared the optimal portfolio weights for a portfolio manager or investor who accounts for systemic jumps in returns and who ignores this feature of the data in Table 5. The portfolio manager might adjust the portfolio weights accounts for systemic jumps, \( \phi \), from that an investor ignores this feature of the data and chooses portfolio weights \( \bar{\phi} \) for pure diffusion. The two portfolios are the same when there are no jumps \( (\lambda = 0) \). Since the optimal portfolio weight \( \Phi^* \) in jump-diffusion model displays positively correlated to the variation \( \Omega \) and intensity of the jump in Equation (9). The empirical results reveal that among five Asian emerging markets, the manager would reduce the portfolio weights from China (0.062), India (0.097), Taiwan (0.249) and Thailand (0.151) and increase the portfolio weight from South Korea (0.010).

<table>
<thead>
<tr>
<th>Countries</th>
<th>Portfolio weights with pure diffusion</th>
<th>Pure diffusion weights on risky ( \phi )</th>
<th>Portfolio weights with jumps diffusion</th>
<th>Jump diffusion weights on risky ( \phi )</th>
<th>Weights difference ( \phi - \bar{\phi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>-0.841</td>
<td>-0.242</td>
<td>-0.903</td>
<td>-0.309</td>
<td>-0.062</td>
</tr>
<tr>
<td>India</td>
<td>2.623</td>
<td>0.756</td>
<td>2.526</td>
<td>0.866</td>
<td>-0.097</td>
</tr>
<tr>
<td>South Korea</td>
<td>1.653</td>
<td>0.477</td>
<td>1.663</td>
<td>0.570</td>
<td>0.010</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1.092</td>
<td>0.315</td>
<td>0.843</td>
<td>0.289</td>
<td>-0.249</td>
</tr>
<tr>
<td>Thailand</td>
<td>-1.060</td>
<td>-0.306</td>
<td>-1.211</td>
<td>-0.415</td>
<td>-0.151</td>
</tr>
</tbody>
</table>

Note: The positive value of \( \phi - \bar{\phi} \) represents the portfolio weights increased and negative value represents the portfolio weights decreased.
Hereafter, in order to calculate the cost of ignoring systemic jumps, we compute the additional wealth needed to raise the expected utility of terminal wealth by comparing the difference of both portfolio policies that considering with jumps or without jumps. In this comparison, we denote by $\Delta \xi$, the additional wealth that makes lifetime expected utility from the portfolio policy that ignores systemic risk compares to that under the optimal policy of considering the rare event jumps.

By calibrating the compensating wealth in the value function, we firstly check the time impact factor on wealth with jump and without jump those are $\kappa = 0.108$ and $\bar{\kappa} = 5.703$, respectively. From the reality of the jump risk incused the portfolio weight adjustment, thus, it reduced the impact of the rare event risk. A compensation of 6.631 for each unit cost of investment can be made or can hedge the loss by the portfolio rearrangement with considering the jump risk.

| Table 6. Investment portfolio weight adjustment of the five emerging Asian markets ($r = 0.02$). |
|---------------------------------|-----------------|-----------------|-----------------|
| Emerging markets               | $\kappa$        | $\bar{\kappa}$ | Risk compensation on wealth $\Delta \xi$ |
| China                          |                 |                 |                 |
| India                          |                 |                 |                 |
| South Korea                    | 0.108           | 5.703           | 6.631           |
| Taiwan                         |                 |                 |                 |
| Thailand                       |                 |                 |                 |

Note: $\kappa$ and $\bar{\kappa}$ are the time impact factors on wealth with and without jump, respectively.

5. Conclusion

Previous work failed to obtain the optimal portfolio weight for two reasons. First, there is still an unknown expected value in the final solution of portfolio weight. Second, the covariance of pure diffusion and covariances of jump diffusion are not identified. Most of the researchers think that the covariance matrix in multiple assets (>2) returns are hard to estimate. For example, Bentzen and Sellin (2003) present an approach to determining the variance but no further results in finding the covariance are obtained because the correlation is not easy to resolve in jump-diffusion model. Table 2 shows a simple result obtained from general statistic estimate, which is quite different from that obtained by stochastic Brownian motion and shown in Table 4.

This paper develops the weight adjustment on rare event jump for investment decisions. This paper investigates the covariance between each two of the five emerging markets using the Monte Carlo Simulation 2000 times and obtains a reasonable value of covariance for jump size. Moreover, this study constructs an optimal portfolio with weighted adjustment and the jump risk consideration in the Asian multinational investment. By comparing a pure diffusion with jump diffusion, the expected returns of assets with the stochastic portfolio equilibrium vector form a new mixed estimate of expected returns. This discussion provides the flexibility to combine the optimal return of the stochastic portfolio by adjusting the weight when the countries suffer any rare event impacts.

This paper provides a comparison of portfolio adjustment by the mixed Poisson jump-diffusion model and the simple Wiener process model of monthly stock return. It concludes that investors may adjust the portfolio weight in international investment to overcome jump
risk and gain risk compensation. The empirical results from the Asian emerging markets show that hedging by adjusting the portfolio weight should be encouraged to reduce the impacts of rare event jumps.

Acknowledgements

The authors would like to acknowledge the financial support provided by the National Science Council in Taiwan under the NSC-96-2416-H-166-003. The authors would like to thank two anonymous referees for suggestions that improved this manuscript. Any remaining errors are ours only.

References

Abramowitz, M., Stegun, I.A. (1972) Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Table, 9th printing. Dover, New York.


317-335.
Accounting, 20 (3), 207-243.
Appendix A

\[ E[e^{AY}] \]

\[ = \int_B \exp(AY) \frac{1}{\sqrt{2\pi}} \frac{1}{\Omega} \exp(-\frac{1}{2} (Y - \theta) \Omega^{-1} (Y - \theta)) Y dY \]

\[ = \int_B \frac{1}{\sqrt{2\pi}} \Omega^{-\frac{1}{2}} \exp(AY - \frac{1}{2} (Y - \theta) \Omega^{-1} (Y - \theta)) Y dY \]

\[ = \int_B \frac{1}{\sqrt{2\pi}} \Omega^{-\frac{1}{2}} \exp[-\frac{1}{2} (Y - (\theta + \Omega A)) \Omega^{-1} (Y - (\theta + \Omega A)) + A'\theta + \frac{1}{2} A' \Omega A] Y dY \]

\[ = \exp(A'\theta + \frac{1}{2} A' \Omega A) \int_B \frac{1}{\sqrt{2\pi}} \Omega^{-\frac{1}{2}} \exp[(Y - (\theta + \Omega A)) \Omega^{-1} (Y - (\theta + \Omega A))] Y dY \]

\[ = \exp(A'\theta + \frac{1}{2} A' \Omega A)(\lambda \theta + \lambda' \Omega \lambda A) \] .......................... (A1)

where \( Y = QX \) and \( Y \sqsubseteq N(\lambda \theta, \lambda' \Omega \lambda A) \), \( \int_B \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} \), \( B \) denotes all the space of \( Y \).

Substitute \( A = -\gamma \Phi \) in Equation (A1)

\[ E[e^{-\gamma \Phi' Y}] = (\theta - \gamma \Phi' \Omega) \exp[-\gamma \Phi' \lambda \theta + \frac{\gamma^2}{2} (\lambda \Phi')' \Omega (\lambda \Phi)] . \]
Appendix B

Let the cumulants $k_n$ be defined by $\ln(\phi(t)) = \sum_{n=1}^{\infty} k_n \frac{(it)^n}{n!}$. Where $\phi(t)$ is the characteristic function, defined as the Fourier transform of the probability density function $P(x)$, using Fourier transform parameters, $\phi(t) = \int_{-\infty}^{\infty} e^{itx} P(x) dx$ (i.e. Abramowitz and Stegun, 1972; Kenney and Keeping, 1951).

Press (1967) assumes that the diffusion component has zero drift ($\mu = 0$), thereby reducing the solution to become four estimated parameters $\hat{\lambda}$, $\hat{\sigma}^2$, $\hat{\delta}^2$ and $\hat{\theta}$ in the model. Moreover, the cumulant matching method relies upon the theoretical relationship between the population cumulants and the parameters of distribution.

Based on the cumulant matching method, Beckers (1981) assumes that the mean jump amplitude is small; therefore, the remaining parameters can be expressed in a simultaneous equation as follows:

$$
\begin{align*}
\hat{\mu} &= \bar{k}_1 \\
\hat{\lambda} &= \frac{25\bar{k}_4}{3\bar{k}_2^2} \\
\hat{\delta}^2 &= \frac{\bar{k}_6}{5\bar{k}_4} \\
\hat{\sigma}^2 &= \bar{k}_2 - \frac{\bar{k}_4^2}{3\bar{k}_6}
\end{align*}
$$

For the reasonable result of the impacts of jump risk, the mean return to the risky asset $\mu$ is independent of the jump occurrence.