The Kinetics of the Stock Markets

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This paper applies the kinematic and kinetic theories of physics to derive two important price behavior equations for the stock markets: the equation of motion and the work-energy equation. Daily data of the Taiwan Stock Exchange index, trading values, and total units of buy and sale orders are used to test the theories. Empirical results indicate that these two equations provide a powerful and good description of the behavior of stock prices. The contribution of this paper is to formally theorize some phenomena that are often heard from the practitioners of stock markets.

Keywords: the equation of motion of the stock prices; the work-energy equation of the stock prices; kinetic energy of stock markets; excess demand; trading values.

1. Introduction

In the field of modern finance, theorists have borrowed some important ideas from physics, such as random walk, chaos, Brownian motion, etc., to describe the behavior of stock prices. Some phenomenal connections between the physical world and the stock markets seem to be interesting and prevailing. In this paper, again, we will borrow the principles of dynamics in physics to explain the behavior of stock prices. The empirical results show that these connections are wonderful and astonishing.

Dynamics is concerned with bodies in motion under the action of forces. Dynamics may subdivide into two branches of study: kinematics and kinetics. Kinematics deals with only the geometry of motion. Specifically, it deals with the mathematical description of motion in terms of position, velocity, and acceleration; whereas, kinetics considers the effects of forces on the motion of bodies. The fundamental properties of force and the relationship between force and acceleration are governed by the Newton’s Laws,1

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1 The Newton’s first law, law of inertia, describes the natural state of motion of a free body on which no external force is acting. Whereas the Newton’s third law, law of action and reaction, says that forces always come in equal and oppositely directed pairs. Forces always occur in pairs; each of them can not exist without the other.
especially the Newton’s Second Law, which relates the external forces acting on a body to its mass and acceleration. This relationship is called the equation of motion. However, when the forces acting on the body can be expressed as functions of space coordinates, the equation of motion is integrated with respect to the displacement of the body. The resulting equation represents the principle of work and kinetic energy.²

Like the prices of other commodities, changes in stock prices are determined by the changes in demand and supply of the stock markets. If there exists an excess demand for a stock, the stock price will go up; conversely, if there exists an excess supply, the stock price will go down. Thus, the relationship between excess demand and price change for stocks is just as the relationship between external force and acceleration for bodies. The first purpose of this paper is to establish the relationship between “forces” acting on stocks and the resulting changes in its stock price by the application of the Newton’s Second Law of motion. This relationship is called the equation of motion for stock markets.

Furthermore, the terminology of “kinetic energy” of the stock markets is often heard in the investment practice. However, it seems to be never investigated, to our knowledge, in the academic literature. The second purpose of this paper is to formally theorize the work-kinetic energy relationship for the stock markets.

In addition to the development of these two important equations for the stock prices, we also conduct an empirical examination to see how well the theories can explain the actual data.

The rest of this paper is organized as follows: The second section is to apply dynamic principle to derive the equation of motion and the equation of work and kinetic energy for the stock prices. The third section describes the empirical methodology. In section four, results are presented and analyzed. Finally, section five is conclusions.

2. Models for the Dynamics of the Stock Markets

The models derived in this paper explicitly direct attention to certain resemblance between the theoretical entities of the dynamics of the stock market and the real physical subject of the dynamics of a body. We will conceptualize the situation as involving the use of analogy. Analogies can lead to the formulation of theories. They are sometimes an utterly essential

² When the forces acting on the stock are expressed as a function of time, the equation of motion is integrated with respect to time and yields the impulse-momentum equation.
part of theories. A better example is perhaps provided by the familiar
dynamic metaphor of particles in financial analysis, with the behavior of
stock prices as random walk, or with the dynamics of stock prices as the
geometric Brownian motion, and so on. No theory is to be condemned as
merely an analogy just because it makes use of one. The point to be
considered is whether or not there is something else to be learned from the
analogy if we do choose to draw it.

The derivation of the models in this paper are based on the following
assumptions:

A1: The dynamic of stock prices for a stock is similar to the dynamic of a
particle subject to external forces. The following analogies are used, with
“shares of a stock” as the “mass of a particle”, the “stock price” as the
“distance” of the movement of a particle, the “change in excess demand
for a stock” as the “external force” exerting on a particle, the “trading
values” of a stock as the “kinetic energy”.

A2: The excess demand for a stock is measurable (or with certainty) rather
than stochastic. In the case of stochastic excess demand for a stock, the
behavior of stock prices is also stochastic.

A3: No price limits are imposed in the stock market.

A4: There is a friction force for a stock, so that the disappearance of the
external force (excess demand) exerting on a stock makes the stock stuck
and the stock price unchanged.

2.1 Kinematics of Stock Markets

Kinematics is the study of the geometry of motion. It deals with the
mathematical description of motion in terms of position, velocity, and
acceleration. Kinematics serves as a prelude to dynamics. In this subsection,
we will concern only with translational motion of stock price, which is
defined as changes in stock price as a function of time. It is convenient to
view a stock as an ideal particle. In this case, prices as a function of time
gives a complete description of the kinematics of stocks.

Let \( S \) be the stock price at time \( t \). The instantaneous velocity, \( V \), of stock
price at a given time is the time rate of change of price at time \( t \). The

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3 An ideal particle is a body with no size and no internal structure. Viewing a stock as an ideal
particle is implicitly defined in financial theories, such as random walk, Brownian motion, etc.
to describe the behavior of stock prices. Although an ideal particle has no size and no internal
structure, it has mass. Similarly, a stock has its “mass”, which is the shares outstanding. When
an external force acts on the stock, the motion of a stock (i.e., price change) will occur.
instantaneous acceleration, \( a \), of stock price is the time rate of change of velocity of stock price at a given time \( t \). The mathematical expressions for velocity and acceleration of stock price are

\[
V = \frac{dS}{dt} \tag{1}
\]

and

\[
a = \frac{dV}{dt} = \frac{d^2 S}{dt^2} \tag{2}
\]

When the velocity \( V \) of the stock price is constant, then the motion is called the uniform motion. Rearranging equation (1), we have

\[
dS = V dt
\]

Integrating both sides, and setting the initial stock price to be \( S_0 \), we obtain

\[
\int_{S_0}^{S} d\zeta = \int_{0}^{t} V d\tau \tag{3}
\]

From this, we have

\[
S = S_0 + V_0 t \tag{4}
\]

When the acceleration, \( a \), of the stock price is constant, then the motion is referred to as the uniform accelerated motion. Rearranging equation (2), we have

\[
dV = adt
\]

Integrating both sides, and setting the initial velocity of stock price to be \( V_0 \), we obtain

\[
\int_{V_0}^{V} dV = \int_{0}^{t} ad\tau \tag{5}
\]

or

\[
V = V_0 + at \tag{6}
\]

In the case of constant acceleration of stock price, the position of stock price can be determined by substituting (6) into (3). That is,

\[
\int_{S_0}^{S} d\zeta = \int_{0}^{t} V d\tau = \int_{0}^{t} (V_0 + a\tau) d\tau
\]
or

\[ S = S_0 + V_0 t + \frac{1}{2} at^2 \]  (7)

The relationship between the velocity \( V \) and the position (price) \( S \), without direct reference to time \( t \), can be obtained as follows: Rearranging equation (2), we write

\[ a = \frac{dV}{dt} = \frac{dV}{dS} \frac{dS}{dt} = V \frac{dV}{dS} \]

or

\[ adS = VdV \]

Integrating both sides, we have

\[ \int_{S_0}^{S} a \, d\zeta = \int_{V_0}^{V} V \, dV \]

\[ a(S - S_0) = \frac{1}{2} (V^2 - V_0^2) \]

or

\[ V^2 = V_0^2 + 2a(S - S_0) \]  (8)

2.2 Kinetics of Stock Markets: Newton’s Second Law of Motion

In this subsection, we will consider the effects of forces on the motion of stock price. We will establish the relationship between the forces acting on the stock and the resulting change in the motion of stock price by directly applying the Newton’s Second Law of motion. This relationship is called the equation of motion.

Like other commodities, stock price is determined by both the demand and supply of the stock market. Equilibrium price is reached when demand equals supply. However, if there exists an excess demand, then the stock price will go up. Conversely, if there exists an excess supply (or negative excess demand), then the stock price will go down. The point is that it is the excess demand causing the stock price to move.\(^4\) This relationship is like the

\(^4\) Of course, there must be some factors (e.g., macroeconomic, industrial, and company-specific factors) affecting a change in excess demand for a stock. However, these intrinsic factors are not our interest in this paper. At this moment, this conclusion is based on the basic economic principles. Later on we will point out that the excess demand at the time of stock market
external forces causing a body to move.

The fundamental properties of force and the relationship between force and acceleration are governed by the Newton’s three laws of motion, especially by the Newton’s Second Law. The Newton’s Second Law establishes the relation between the force, \( F \), acting on a body and acceleration, \( a \), caused by this force. Mathematical expression of the Newton’s Second Law is as follows:

\[
a = \frac{F}{m}
\]

where \( m \) is the mass of the body. Newton’s Second Law is usually called the equation of motion.

In stock markets, the excess demand acts as the external force that causes the stock price to change. In this paper, the excess demand, \( ED \), is defined as the total units of buy orders minus the total units of sale orders for a stock. Since some buy and sale orders are ineffective orders,\(^5\) the excess demand at the time of stock market opening in each trading day usually provides little explanation power. This means that a negative excess demand at the opening of stock market does not necessarily imply that the stock price will go down. However, changes in excess demand in the subsequent trading time do provide explanatory power.\(^6\) An increasing excess demand will follow an up of the stock price, and vice versa. From this discussion, change in excess demand, \( \Delta ED \), will be used hereafter as external force acting on the stock price. Thus, according to the Newton's Second Law of motion, we obtain the following equation of motion for the stock markets.

\[
a = \frac{1}{m} \Delta ED
\]

where \( a \) is the acceleration of stock price, and \( m \) is the “mass” of the stock.

Equation (10) states that the acceleration of stock price is proportional to the change in excess demand and inversely proportional to the “mass” of

\(^5\) A buy order is not an effective order if the price specified by the order is lower than the prevailing market price. Similarly, a sale order is not an effective order if the price specified by the order is higher than the prevailing market price.

\(^6\) Again, part of changes in excess demand are ineffective and part of changes in excess demand are effective. We assume that the ratio of effective changes in excess demand to ineffective changes in excess demand keeps constant.
the stock. Thus, other thing being equal, we expect that the more the change in excess demand, the larger the acceleration of stock price; the smaller the size of the stock, the larger the acceleration of stock price.

2.3 Kinetics of Stock Markets: Work and Energy

When the force acting on the stock can be expressed as functions of space coordinates, the equation of motion for the stock market is integrated with respect to the displacement of the stock price. The resulting equation represents the principle of work and kinetic energy for the stock markets.

The work done by a force on the stock price is defined as the force times the displacement of the stock price in the direction of the force. The expression for the infinitesimal work, \( dW \), done by the force, \( F \), on the stock during its infinitesimal displacement, \( dS \), is defined as

\[
dW = FdS
\] (11)

The total work \( W \) done by the force \( F \) on the stock price, as it moves between points A and B is defined as

\[
W = \int_{A}^{B} FdS
\] (12)

Rewrite the Newton’s Second Law of motion as

\[
F = ma = m \frac{dV}{dt}
\] (13)

Substituting equation (13) into equation (12), yields

\[
W = \int_{t_A}^{t_B} m \frac{dV}{dt} \cdot V \, dt = \int_{t_A}^{t_B} mV \cdot dV
\]
or

\[
W = \frac{1}{2} mV_B^2 - \frac{1}{2} mV_A^2
\] (14)

Equation (14) is the equation of work and kinetic energy for the stock markets. Note that \( \frac{1}{2} mV^2 \) is the kinetic energy (KE). The equation (14) says that the work done on the stock price is equal to the change in kinetic energy, or

\[
\Delta KE = \int_{A}^{B} FdS
\] (15)
If $F$ is constant, then equation (15) becomes

$$\Delta KE = F(S_B - S_A) = F \Delta S$$  \hspace{1cm} (16)

Substituting $\Delta ED$ for $F$ in equation (16), yields

$$\Delta KE = \Delta ED \times \Delta S$$

Usually, $\Delta ED$ is proportional to $\Delta S$, and has the same direction as $\Delta S$. Therefore, an increase of the change in excess demand will follow an increase of the kinetic energy of the stock markets; and vise versa.

2.4 Kinetics of Stock Markets: Impulse and Momentum

When the forces acting on the stock are expressed as a function of time, the equation of motion for the stock market is integrated with respect to time and yields the impulse-momentum equation for the stock markets.

Assuming that the “mass” of a stock is constant, equation (13) can be arranged to read

$$F = \frac{d}{dt}(mV)$$  \hspace{1cm} (17)

where $mV$ is called the momentum of the stock markets. This equation states that the resultant force acting on a stock is equal to the time rate of change of the momentum. Multiplying both sides of equation (17) by $dt$ and integrating between time $t_1$ and $t_2$, yields

$$\int_{t_1}^{t_2} Fd\tau = \int_{V_1}^{V_2} d(mV) = mV_2 - mV_1$$  \hspace{1cm} (18)

The integral $\int_{t_1}^{t_2} Fdt$ is called the impulse acting on the stock. Equation (18) states that the change of momentum of the stock market equals the impulse of the external forces.

3. Empirical Methodology

3.1 The Data

Because the data for buy and sale orders of individual stocks are unavailable, we use the Taiwan Stock Exchange (TSE) index to implement an empirical study. Variables in this study include daily trading values of the TSE, total units of buy orders, total units of sale orders (1 unit = 1,000
shares), and the indexes. The daily data for trading values and the TSE indexes are collected from the “EPS/AREMOS” data base. The daily data for total units of buy orders and total units of sale orders for all stocks listing in TSE are obtained from the Taiwan Stock Exchange. The data covers a seven-year period, December 28, 1989 - March 31, 1998. A total of 2,350 trading days are included.

3.2 Empirical Methodology

For the practical interest, we will test two principles: the equation of motion for the stock markets and equation of work and kinetic energy for the stock markets.

A. Test of the equation of motion for the stock markets

In the previous section, we mention that the relation of change in excess demand to the acceleration of stock prices is just as the relation of external forces to the acceleration of a body. The excess demand for stocks at time \( \tau \) is defined as the difference between the total units of buy orders at \( \tau \) and the total units of sale orders at \( \tau \). i.e.,

\[
ED(T, \tau) = TB(T, \tau) - TS(T, \tau)
\]

where \( ED(T, \tau) \) represents the excess demand at time \( \tau \) on date \( T \), \( TB(T, \tau) \) represents the total units of buy orders at time \( \tau \) on date \( T \), and \( TS(T, \tau) \) represents the total units of sale orders at time \( \tau \) on date \( T \). Next, the change in excess demand on date \( T \) is defined as the difference in the excess demand between the opening time of the stock market and the close time of the stock market on date \( T \). That is

\[
ED(T, 9:00am) = TB(T, 9:00am) - TS(T, 9:00am)
\]

\[
ED(T, 12:00noon) = TB(T, 12:00noon) - TS(T, 12:00noon)
\]

and

\[
\Delta ED(T) = ED(T, 12:00noon) - ED(T, 9:00noon)
\]

Like other commodities, stock price is determined by the change in demand and supply of the stock market. If there exists a change in excess demand, then the stock price will go up. Conversely, if there exists a change in excess supply, then the stock price will go down. Let \( \Delta S(T) \) represent the change in stock price between date \( T \) and date \( T-1 \), i.e.,

\[
\Delta S(T) = S(T) - S(T-1)
\]

From equation (7), and noting that \( V_0 = 0 \) and \( a = 0 \) at the opening time of a
stock market in each trading day, we obtain the price change in stock at date \( t \) as follows:

\[
S_t - S_{t-1} = 0 + \int_0^1 a_t \tau \, d\tau
\]

or

\[
\Delta S_t = \frac{1}{2} a_t \times 1^2
\]  

(22)

where \( a_t \) is the acceleration of stock price at date \( t \).

Substituting the Newton’s Second Law (equation (9)) into (22), yields

\[
\Delta S(T) = \frac{1}{2} \frac{\Delta ED(T)}{m}
\]

(23)

To test the equation of motion for the stock markets, equation (23), the following regression equation is used:

\[
\Delta S_t = \alpha + \beta \Delta ED_t + u_t
\]

(24)

If the theory is correct, then \( \alpha = 0 \), and \( \beta \neq 0 \). Because the data for excess demand of individual stocks are unavailable, we use the aggregate of total excess demand data for all stocks in TSE to test this principle and prediction.

B. Test of the Equation of Work and Kinetic Energy for the Stock Markets

The principle of work and kinetic energy for the stock markets states that the work done by external force (i.e., change in excess demand) on the stock market is equal to the change in kinetic energy. Firstly, the work done by the external force at date \( T \) is

\[
W(T) = \Delta ED(T) \times \Delta S(T)
\]

(25)

Next, the total trading value of TSE at date \( T \) is used as a surrogate of the kinetic energy of stock market. The daily trading value is also the change in kinetic energy at date \( T \) because the trading value at the open of stock market is zero. Therefore, the equation of work and kinetic energy for the

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7 Equation (7) represents the case of continuous motion. However, in the case of stock market, stock price changes start from the opening of a stock market in each trading day. Therefore, at the opening of the stock market, \( V(t, 9:00am) = V_0 = 0 \) and \( a = 0 \).

8 From equation (7), it seems that there are two factors, \( \Delta CD(T) \) and \( m \), affecting the stock price change, \( \Delta S(T) \). However, in the empirical test, we only set one explanatory variable, \( \Delta CD \), in regression (24) for the following reasons: one is the fact that the total mass of the whole stock market (i.e., the total shares outstanding of stocks listed in the TSE) in a short period keeps almost unchanged. The other is that the magnitude of will reflect the magnitude of \( m \). is inversely related to \( m \).
stock market is

$$\Delta KE(T) = \Delta ED(T) \times \Delta S(T)$$  \hspace{1cm} (26)

To test the adequacy of the equation of work and kinetic energy, the following regression equation is applied

$$\ln(\Delta KE_i) = \alpha + \beta \ln|\Delta ED_i| + \gamma \ln|\Delta S_i| + \epsilon_i$$  \hspace{1cm} (27)

The absolute values of $\Delta ED$ and $\Delta S$ is required and correct because: (a) the definition of logarithmic functions requires nonnegative values as its domain, and (b) $\Delta ED$ and $\Delta S$ are in the same directions (i.e., same signs). If the trading value at date $T$ can truly represent the change in kinetic energy, then, in theory, $\alpha = 0$, $\beta = 1$, and $\gamma = 1$. However, if the trading value at date $T$ is only a proxy, then $\beta$ and $\gamma$ are not necessarily equal to 1.

C. Econometric considerations

Because regression equations (24) and (27) are used for testing the adequacy of the two theories. The time series of data may violate the assumptions of regression equations. Thus, the following steps are implemented for better estimation and testing.

1. Test of the stability of regression parameters

   Because the data covers a long period of time, the parameters of regression equations may change. To consider this possibility, the dummy variable technique is used for estimation in different market conditions (bull and bear). For this purpose, the whole sample period is divided into three subperiods by two time points, October 1, 1990 and July 31, 1997, on which a bull market and a bear market are roughly divided. The first and third periods belong to bear markets and the second period is a bull market.

2. Test of homoscedasticity

   Because trading values and stock volatilities in different market conditions are likely to be different, a test of homoscedasticity of regression residuals is needed. The Glejser method\(^9\) is used for this purpose.

3. Test of autocorrelation

   To detect the autocorrelation of regression equation, the Durbin-
Watson test is used. If presence of autocorrelation is found, then the Cochrane-Orcutt procedure is used for estimation.

4. Unit-Root Test

To test whether the time series data is stationary for regression, the augmented Dickey-Fuller (ADF) test is used. If the data exists an unit root, then the data is not stationary and the difference-stationary procedure is used.

5. OLS vs. GARCH Estimation

Finally, if the regression residuals show an existence of autoregressive conditional heteroscedasticity (ARCH), then the GARCH method of Bollerslve (1986) is used for the estimation; otherwise, OLS method is used for estimation.

4. Results and Analysis

4.1 Test of the Equation of Motion for the Stock Markets

For convenience of reading, we repeat the equation of motion for the stock markets as

\[ \Delta S_t = \alpha + \beta \Delta ED_t + u_t \] (24)

Table 1 shows the regression results for the estimation of the coefficients of the equation of motion. In general, all \( \beta \) s are highly statistically significant at .005 level (or better) for the whole period and each subperiods. Adjusted R-square shows that the variability of \( \Delta ED \) (changes in excess demand) could explain about 40% variability of \( \Delta S \) (stock price change). This result is consistent with Hsu [5]. Values of \( \beta \) s range from .000203 to .00056 with average value of about .00034. This means that an unit increase in excess demand will raise the TSE index about .00034 points.

Since the values of \( \beta \) in different market conditions (bull and bear) and in different subperiods are different, the effects of one unit change in excess demand on the change in TSE index in different periods are different. It is interesting to mention that the effect of one unit change in excess demand in period 1990/01/04 - 1990/10/01 is twice more than those of other periods. This result can be explained as follows: First, changes in excess demand for stocks in different periods are likely to differ because of different business cycles for individual industries and individual stocks. Therefore,
some stocks are likely to be actively traded in one period and other stocks are likely to be actively traded in another periods. Second, the "masses" of individual stocks are different, and the data for excess demand are aggregate data. If the distribution of percentages of change in excess demand among individual stocks varies over time, then $\beta$ should be changed over time. There is still a third explanation that the friction in different market trends is different. For example, in a trend of sharply rising in stock price, bid prices in buy orders are likely to be much higher than the prevailing market price. On the other hand, in a trend of sharply sliding in stock price, offer prices in sale orders are likely to be much lower than the prevailing market price. Thus, the sensitivity of stock price change to the change in excess demand in different market trends is different.\footnote{We thank an anonymous referee for pointing out this explanation.}

Table 1  Summary for the Regressional Results for the Equation of Motion for Stock Market (Unrestricted)

<table>
<thead>
<tr>
<th>Periods</th>
<th>Regression Coefficients</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>The Whole Period</td>
<td>90/01/04-98/03/31</td>
<td>-1.684842 (0.896909)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-26.35779 (-2.889183)***</td>
</tr>
<tr>
<td></td>
<td>Subperiods</td>
<td>90/01/04-90/12/27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>91/01/03-91/12/28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>92/01/04-92/12/29</td>
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<tr>
<td></td>
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<td>93/01/05-93/12/31</td>
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<tr>
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<td></td>
<td>96/01/04-96/12/31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>97/01/04-98/03/31</td>
</tr>
</tbody>
</table>

|                  | Bear-market Period      | 90/01/04-90/10/01     | -28.06827 (-2.556446)* | 0.000645 (15.73251)*** | 0.540996 |
|                  | Bull-market Period      | 90/10/02-97/07/31     | 2.153219 (1.340060) | 0.000273 (32.28808)*** | 0.347910 |
|                  | Bear-market Period      | 97/08/01-98/03/31     | -5.155517 (-0.613512) | 0.000308 (9.90584)*** | 0.354084 |

Note: 1. Figures in parentheses ( ) are t-values
2. * significant at 0.05, ** significant at 0.01, *** significant at 0.005 or better.
In theorem, regression equation (24) should have no intercept (by the Newton's Second Law). From table 1, for the whole period, $\alpha$ is not statistically significant. For the three bull/bear periods, only the first period has significant $\alpha$. However, for some years, $\alpha$s are statistically significant. Table 2 shows the results for the regression with restriction on $\alpha (=0)$. From tables 1 and 2, it is seen that the corresponding $\beta$s are almost the same regardless the regression is restricted or not.

**Table 2** Summary for the Regressional Results for the Equation of Motion for Stock Markets (Restricted)

<table>
<thead>
<tr>
<th>Periods</th>
<th>$\beta$</th>
<th>t-value</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Whole Period</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subperiods</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>90/01-98/03/31</td>
<td>0.000336</td>
<td>36.69770***</td>
<td>0.364496</td>
</tr>
<tr>
<td>90/01-90/12/27</td>
<td>0.000556</td>
<td>17.14270***</td>
<td>0.508784</td>
</tr>
<tr>
<td>91/01-91/12/28</td>
<td>0.000394</td>
<td>10.55893***</td>
<td>0.281191</td>
</tr>
<tr>
<td>92/01-92/12/29</td>
<td>0.000324</td>
<td>9.93272***</td>
<td>0.254542</td>
</tr>
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<td>93/01-93/12/31</td>
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<td>15.15139***</td>
<td>0.433324</td>
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<td>14.36247***</td>
<td>0.418981</td>
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<td>95/01-95/12/30</td>
<td>0.000237</td>
<td>10.11090***</td>
<td>0.257807</td>
</tr>
<tr>
<td>96/01-96/12/31</td>
<td>0.000202</td>
<td>11.08025***</td>
<td>0.293841</td>
</tr>
<tr>
<td>97/01-98/03/31</td>
<td>0.000250</td>
<td>14.24386***</td>
<td>0.367347</td>
</tr>
<tr>
<td>Bear-market Period</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90/01-90/10/01</td>
<td>0.000648</td>
<td>15.61406***</td>
<td>0.526711</td>
</tr>
<tr>
<td>Bull-market Period</td>
<td>90/10-97/07/31</td>
<td>0.000274</td>
<td>32.34204***</td>
</tr>
<tr>
<td>Bear-market Period</td>
<td>97/08-98/03/31</td>
<td>0.000309</td>
<td>9.92480***</td>
</tr>
</tbody>
</table>

Note: * significant at 0.05, ** significant at 0.01, *** significant at 0.005 or better.

A. Model Modification

1. Test of the stability of regression parameters

Dummy variable technique is used to test the stability of regression coefficients as follows:

$$\Delta S_t = \alpha + \beta \Delta ED_t + \gamma D_1 + \delta D_2 + \zeta D_1 \Delta ED_t + \eta D_2 \Delta ED_t + U_t,$$

\[
D_1 = \begin{cases} 
1 & \text{for } 90/10/02 - 97/07/31 \\
0 & \text{otherwise}
\end{cases}
\]

\[
D_2 = \begin{cases} 
1 & \text{for } 97/08/01 - 98/03/31 \\
0 & \text{otherwise}
\end{cases}
\]
The estimated regression equation is

\[
\Delta S_t = -28.068 + 0.000645 \Delta ED_t + 30.221D_1 + 22.913D_2 \\
(-4.726)*** (29.082)*** (4.833)*** (2.619)**
\]

\[-0.000372D_1 \Delta ED_t - 0.00036D_2 \Delta ED_t \]

\[R^2=0.428\]

From above equation, it is seen that a structural change exists. Therefore, the estimated regression equations for the three bull/bear markets should be different. The results are as follows:

For period 1990/01/01-1990/10/31

\[\Delta S_{t1} = -28.068 + 0.000645 \Delta ED_{t1}\]

For period 1990/10/02-1997/07/31

\[\Delta S_{t2} = 2.153 + 0.000273 \Delta ED_{t2}\]

For period 1997/08/01-1998/03/31

\[\Delta S_{t3} = -5.155 + 0.000309 \Delta ED_{t3}\]

2. Test of heteroscedasticity

Using Glejser method for testing heteroscedasticity of the above regression residuals, \(U_t\). That is,

\[U_t = \alpha + \beta \cdot \Delta ED_t + \mu_t\]

H_0 : \(\beta = 0\)

The estimated results are

\[|U_{t1}| = 119.109 - 0.000056 \cdot \Delta ED_{t1} \quad \text{heteroscedasticity}\]

\[(16.623)*** (-2.092)*\]

\[|U_{t2}| = 51.891 - 0.00000349 \cdot \Delta ED_{t2} \quad \text{homoedasticity}\]

\[(47.305)*** (-0.603)\]

\[|U_{t3}| = 87.037 + 0.0000243 \cdot \Delta ED_{t3} \quad \text{homoedasticity}\]

\[(87.037)*** (1.235)\]
1. Figures in parentheses ( ) are t-values

2. *: significant at 0.05, **: significant at 0.01, ***: significant at 0.005 or better

From these results, we conclude that heteroscedasticity is existed in the first period

3. Test of autocorrelation

First, Durbin-Watson test is applied to the above regression. The D-W values for the three subperiods are 1.595, 1.822, and 1.982, respectively. Then, the Cochrane-Orcutt procedure is used to correct autocorrelation. The final D-W values are 2.01, 2.004, and 1.981, respectively. This shows that autocorrelation for the three regression no longer exist. Finally, the Glejser method is applied again, the heteroscedasticity also disappears.

4. Unit-root test

Since all ADF-t values are greater than their Mackinnon critical values, the null hypothesis that the time series data has unit root is rejected. Therefore, the data for regression is stationary.

B. Price Prediction Using the Equation of Motion

First, four time-length periods (1-year, 2-year, 3-year, and 4-year periods) of data are used for estimating the parameters of the equation of motion for the stock markets (Eq.(24)) to obtain four estimated regression equations. Then, each estimated regression is used to predict the daily stock indexes of four subsequent periods (1-month, 2-month, 3-month, and 4-month lengths), respectively. For each estimation period and its subsequent prediction period, the process is rolling over. For example, for one-year estimation period and one-month prediction period, period January 1990-December 1990 data are first used for estimation of regression function. Then daily stock indexes in January 1991 are predicted. After this, period February 1990-January 1991 data are used for estimation and then daily stock indexes in February 1991 are predicted... and so on. The predicted values are compared with the actual stock prices, and then percentage of prediction errors (= (actual index – predicted index)/ predicted index) are calculated.

Table 3 reports the results of prediction errors (in percentage) by using OLS. Panels A through D separate the results for the four estimation periods. Columns 2 through 5 give the results for the four prediction periods. From Table 3, we find that maximum prediction errors range from .48% (for the 3-year estimation period, 6-month prediction period) to 1.32% (for the 3-year estimation period, 1-month prediction period). Average prediction errors
range from -.0006% (for the 4-year estimation period, 1-month prediction period) to .0501% (for the 3-year estimation period, 2-month prediction period). All prediction errors are not statistically significantly different from zero.

In general, the longer the estimation period and the shorter the prediction period, the more accurate for the prediction of stock prices. Using the data of 4 years to estimate the equation of motion for the stock markets gives the best prediction (prediction errors range from – .0006% to – .0076%).

<table>
<thead>
<tr>
<th>Prediction Period</th>
<th>One Month</th>
<th>Two Month</th>
<th>Three Month</th>
<th>Six Month</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: One-Year Estimation Period</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>1.194%</td>
<td>1.053%</td>
<td>1.040%</td>
<td>0.766%</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.001%</td>
<td>0.008%</td>
<td>-0.018%</td>
<td>0.025%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0348%</td>
<td>0.0403%</td>
<td>0.0413%</td>
<td>0.0430%</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0044</td>
<td>0.0035</td>
<td>0.0032</td>
<td>0.0027</td>
</tr>
<tr>
<td>t-value</td>
<td>0.0053</td>
<td>0.0068</td>
<td>0.0073</td>
<td>0.0083</td>
</tr>
<tr>
<td>p-value</td>
<td>0.9958</td>
<td>0.9946</td>
<td>0.9942</td>
<td>0.9934</td>
</tr>
<tr>
<td><strong>Panel B: Two-Year Estimation Period</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>1.068%</td>
<td>0.807%</td>
<td>0.618%</td>
<td>-0.524%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.003%</td>
<td>-0.005%</td>
<td>0.028%</td>
<td>-0.001%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0320%</td>
<td>0.0333%</td>
<td>0.0300%</td>
<td>0.0302%</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0042</td>
<td>0.0034</td>
<td>0.0030</td>
<td>0.0025</td>
</tr>
<tr>
<td>t-value</td>
<td>0.0050</td>
<td>0.0057</td>
<td>0.0055</td>
<td>0.0060</td>
</tr>
<tr>
<td>p-value</td>
<td>0.9961</td>
<td>0.9955</td>
<td>0.9956</td>
<td>0.9952</td>
</tr>
<tr>
<td><strong>Panel C: Three-Year Estimation Period</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>1.324%</td>
<td>0.950%</td>
<td>0.787%</td>
<td>0.477%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.006%</td>
<td>0.029%</td>
<td>0.001%</td>
<td>-0.009%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0436%</td>
<td>0.0501%</td>
<td>0.0447%</td>
<td>0.0350%</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0041</td>
<td>0.0035</td>
<td>0.0031</td>
<td>0.0026</td>
</tr>
<tr>
<td>t-value</td>
<td>0.0068</td>
<td>0.0085</td>
<td>0.0081</td>
<td>0.0068</td>
</tr>
<tr>
<td>p-value</td>
<td>0.9946</td>
<td>0.9932</td>
<td>0.9936</td>
<td>0.9946</td>
</tr>
<tr>
<td><strong>Panel D: Four-Year Estimation Period</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum</td>
<td>0.948%</td>
<td>0.767%</td>
<td>0.688%</td>
<td>0.667%</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.014%</td>
<td>-0.003%</td>
<td>0.001%</td>
<td>-0.015%</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0006%</td>
<td>-0.0010%</td>
<td>-0.0016%</td>
<td>-0.0076%</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0039</td>
<td>0.0033</td>
<td>0.0030</td>
<td>0.0027</td>
</tr>
<tr>
<td>t-value</td>
<td>-0.0001</td>
<td>-0.0002</td>
<td>-0.0003</td>
<td>-0.0015</td>
</tr>
<tr>
<td>p-value</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9998</td>
<td>0.9988</td>
</tr>
</tbody>
</table>
Table 4 gives the results of prediction errors by using GARCH. In general, using GARCH model to estimate gives more accurate results than by using OLS method. However, for the GARCH method, the 2-year estimation period gives the best results. In this case, the longer the prediction period, the more accurate for the prediction.

Table 4  Prediction Errors of the Equation of Motion for Stock Markets (GARCH Method)

<table>
<thead>
<tr>
<th>Prediction Period</th>
<th>One-Month</th>
<th>Two-Month</th>
<th>Three-Month</th>
<th>Six-Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>1.169%</td>
<td>1.048%</td>
<td>1.035%</td>
<td>0.761%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.004%</td>
<td>0.006%</td>
<td>0.007%</td>
<td>-0.022%</td>
</tr>
<tr>
<td>Mean</td>
<td>0.023%</td>
<td>0.028%</td>
<td>0.030%</td>
<td>0.032%</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0044</td>
<td>0.0036</td>
<td>0.0033</td>
<td>0.0029</td>
</tr>
<tr>
<td>t-value</td>
<td>0.0034</td>
<td>0.0047</td>
<td>0.0052</td>
<td>0.0059</td>
</tr>
<tr>
<td>p-value</td>
<td>0.9973</td>
<td>0.9962</td>
<td>0.9959</td>
<td>0.9953</td>
</tr>
</tbody>
</table>

Panel A: One-Year Estimation Period

Max: 1.169% 1.048% 1.035% 0.761%
Min: -0.004% 0.006% 0.007% -0.022%
Mean: 0.023% 0.028% 0.030% 0.032%
Var: 0.0044 0.0036 0.0033 0.0029
t: 0.0034 0.0047 0.0052 0.0059
p: 0.9973 0.9962 0.9959 0.9953

Panel B: Two-Year Estimation Period

Max: 1.054% 0.695% 0.586% 0.440%
Min: 0.016% -0.004% 0.001% -0.003%
Mean: 0.0017% 0.0021% 0.0004% 0.0008%
Var: 0.0041 0.0033 0.0029 0.0024
t: 0.0003 0.0004 0.0001 0.0002
p: 0.9998 0.9997 0.9999 0.9999

Panel C: Three-Year Estimation Period

Max: 1.143% 0.779% 0.593% -0.466%
Min: 0.013% 0.016% -0.001% -0.008%
Mean: 0.0168% 0.0233% 0.0179% 0.0072%
Var: 0.0039 0.0032 0.0028 0.0023
t: 0.0027 0.0041 0.0034 0.0015
p: 0.9979 0.9967 0.9973 0.9988

Panel D: Four-Year Estimation Period

Max: 0.776% -0.617% 0.492% 0.461%
Min: 0.013% 0.016% 0.000% -0.010%
Mean: -0.0281% -0.0281% -0.0283% -0.0337%
Var: 0.0037 0.0030 0.0027 0.0023
t: -0.0046 -0.0051 -0.0054 -0.0070
p: 0.9963 0.9959 0.9957 0.9944

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4.2 Test of Equation of Work and Kinetic Energy

For convenience of reading, we repeat the equation of work and energy as follows

\[
\ln(\Delta KE_t) = a + b \cdot \ln|\Delta ED_t| + c \cdot \ln|\Delta S_t| + \varepsilon_t
\]  

(27)

Table 5 shows the preliminary results of the above regression, which seem not to give a satisfactory explanation because of low \(R^2\) s (ranging from .034 to .24). A further model modification is as follows:

<table>
<thead>
<tr>
<th>Periods</th>
<th>Regression Coefficients</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>The Whole Period:&lt;br&gt;90/01/04-98/03/31</td>
<td>3.451155</td>
<td>0.190378</td>
</tr>
<tr>
<td></td>
<td>(53.42002)***</td>
<td>(14.22132)***</td>
</tr>
<tr>
<td>Subperiods:&lt;br&gt;90/01/04-90/12/27</td>
<td>3.983761</td>
<td>0.136507</td>
</tr>
<tr>
<td></td>
<td>(21.12596)***</td>
<td>(3.642377)***</td>
</tr>
<tr>
<td>91/01/03-91/12/28</td>
<td>3.988823</td>
<td>0.054423</td>
</tr>
<tr>
<td></td>
<td>(26.80048)***</td>
<td>(1.813424)***</td>
</tr>
<tr>
<td>92/01/04-92/12/29</td>
<td>3.574796</td>
<td>0.119877</td>
</tr>
<tr>
<td></td>
<td>(22.33627)***</td>
<td>(3.546553)***</td>
</tr>
<tr>
<td>93/01/05-93/12/31</td>
<td>3.083459</td>
<td>0.226792</td>
</tr>
<tr>
<td></td>
<td>(18.44636)***</td>
<td>(6.465044)***</td>
</tr>
<tr>
<td>94/01/05-94/12/31</td>
<td>4.288767</td>
<td>0.084951</td>
</tr>
<tr>
<td></td>
<td>(37.25287)***</td>
<td>(3.676574)***</td>
</tr>
<tr>
<td>95/01/05-95/12/30</td>
<td>4.273281</td>
<td>0.038326</td>
</tr>
<tr>
<td></td>
<td>(43.36877)***</td>
<td>(1.902564)***</td>
</tr>
<tr>
<td>96/01/04-96/12/31</td>
<td>4.204234</td>
<td>0.053622</td>
</tr>
<tr>
<td></td>
<td>(35.34179)***</td>
<td>(2.218882)*</td>
</tr>
<tr>
<td>97/01/04-98/03/31</td>
<td>4.806462</td>
<td>0.052215</td>
</tr>
<tr>
<td></td>
<td>(56.19520)***</td>
<td>(3.104861)***</td>
</tr>
<tr>
<td>Bear-market Period:&lt;br&gt;90/01/04-90/10/01</td>
<td>4.028037</td>
<td>0.165505</td>
</tr>
<tr>
<td></td>
<td>(18.94134)***</td>
<td>(3.915833)***</td>
</tr>
<tr>
<td>Bull-market Period:&lt;br&gt;90/10/02-97/07/31</td>
<td>3.452528</td>
<td>0.188516</td>
</tr>
<tr>
<td></td>
<td>(51.25088)***</td>
<td>(13.56387)***</td>
</tr>
<tr>
<td>Bear-market Period:&lt;br&gt;97/08/01-98/03/31</td>
<td>4.809284</td>
<td>0.059616</td>
</tr>
<tr>
<td></td>
<td>(44.13272)***</td>
<td>(2.770941)**</td>
</tr>
</tbody>
</table>

Note: 1. Figures in parentheses ( ) are t-values
2. * significant at 0.05, ** significant at 0.01, *** significant at 0.005 or better
A. Model Modification

1. Test of the stability of regression parameters

Dummy variable technique is used to test the stability of regression coefficients as follows:

\[
\ln(\Delta K_{E_i}) = a + b \cdot \ln|\Delta ED_i| + c \cdot \ln|\Delta S_i| + dD_1 + eD_2 + fD_1 \cdot \ln|\Delta ED_i| + gD_2 \cdot \ln|\Delta ED_i| + hD_1 \cdot |\Delta S_i| + iD_2 \cdot \ln|\Delta S_i| + \theta_i
\]

\[
D_1 = \begin{cases} 
1 & \text{for 90/10/02 – 97/07/31} \\
0 & \text{otherwise}
\end{cases}
\]

\[
D_2 = \begin{cases} 
1 & \text{for 97/08/01 – 98/03/31} \\
0 & \text{otherwise}
\end{cases}
\]

The estimated regression equation is

\[
\ln(\Delta K_{E_i}) = 4.028 + 0.165505 \cdot \ln|\Delta ED_i| - 0.037479 \cdot \ln|\Delta S_i| - 0.575508 \cdot D_1
\]

\[
(18.645)*** \quad (3.855)*** \quad (-0.841) \quad (-2.550)^*
\]

\[
+ 0.781247 \cdot D_2 + 0.023011 \cdot D_1 \ln|\Delta ED_i| - 0.105889 \cdot D_2 \ln|\Delta ED_i|
\]

\[
(2.613)** \quad (0.511) \quad (-1.788)
\]

\[
+ 0.158174 \cdot D_1 \ln|\Delta S_i| + 0.032087 \cdot D_2 \ln|\Delta S_i|
\]

\[
(3.421)^*** \quad (0.502)
\]

Note: 1. Figures in parentheses ( ) are t-values

2. * significant at 0.05, ** significant at 0.01, *** significant at 0.005 or better

From above equation, it is seen that a structural change exists. Therefore, the estimated regression equations for the three bull/bear markets should be different. The results are as follows:

For period 1990/01/01-1990/10/31,

\[
\ln(\Delta K_{E_i}) = 4.028 + 0.165505 \cdot \ln|\Delta ED_{i1}| - 0.037479 \cdot \ln|\Delta S_{i1}|
\]

For period 1990/10/02-1997/07/31,

\[
\ln(\Delta K_{E_i}) = 3.452492 + 0.188516 \cdot \ln|\Delta ED_{i2}| + 0.120695 \cdot \ln|\Delta S_{i2}|
\]
For period 1997/08/01-1998/03/31,

\[
\ln(\Delta KE_{t,3}) = 4.809247 + 0.059616 \cdot \ln|\Delta ED_{t,3}| + 0.005392 \cdot \ln|\Delta S_{t,3}|
\]

2. Test of heteroscedasticity

Using Glejser method for testing heteroscedasticity of the above regression residuals, \( \theta_t \). That is,

\[
|\theta_t| = a + b \cdot \ln|\Delta ED_t| + c \cdot \ln|\Delta S_t| + \epsilon_t
\]

\[H_0: a = b = 0 \] i.e., homoscedasticity

The estimated results are

\[
|\theta_{t1}| = 0.436 - 0.046842 \cdot \ln|\Delta ED_{t1}| + 0.015557 \cdot \ln|\Delta S_{t1}| \quad \text{homoscedasticity}
\]

(3.566)*** (-1.926) (0.616)

\[
|\theta_{t2}| = 51.891 - 0.00000349 \cdot \ln|\Delta ED_{t2}| - 0.020982 \cdot \ln|\Delta S_{t2}| \quad \text{heteroscedasticity}
\]

(10.084)*** (-3.265)*** (-2.803)**

\[
|\theta_{t3}| = 87.037 + 0.0000243 \cdot \ln|\Delta ED_{t3}| + 0.004208 \cdot \ln|\Delta S_{t3}| \quad \text{homoscedasticity}
\]

(3.681)*** (-1.537) (0.338)

Note: 1. Figures in parentheses ( ) are t-values
2. * significant at 0.05, ** significant at 0.01, *** significant at 0.005 or better

From these results, we conclude that heteroscedasticity is existed in the second period.

3. Test of autocorrelation

First, Durbin-Watson test is applied to the above regression. The D-W values for the three subperiods are .354, .326, and .472, respectively. Then, the Cochrane-Orcutt procedure is used to correct autocorrelation. The final D-W values are 2.164, 2.412, and 2.243, respectively. This shows that autocorrelation for the three regression no longer exist. Finally, the Glejser method is applied again, the heteroscedasticity also disappears.

4. Unit-root test

Since all ADF-t values are smaller than their Mackinnon critical values, the null hypothesis that the time series data has unit root is accepted. Therefore, the data for regression is none stationary. This implies the first
differences for the data should be taken.

B. Results and Analysis

After the modification and adjustment of the original regression equation, though there still exists an indecisive autocorrelation in the second period regression, the basic assumptions regarding all the regression equations are, in general, acceptable. Furthermore, after taking the first-order difference of the regression variables, the data for regression is stationary and the explanation powers for all regressions are highly improved (with adjusted $R^2 = .90$ for the whole period). Table 6 shows the regression results for the equation of work and kinetic energy after taking the first-order difference of regression variables. A high explanation power probably implies that the correct work-energy principle for the stock markets is that the variables should be expressed as percentages. That is,

$$\ln(\Delta KE_t) - \ln(\Delta KE_{t-1}) = a + b(\ln|\Delta ED_t| - \ln|\Delta ED_{t-1}|) + c(\ln|\Delta S_t| - \ln|\Delta S_{t-1}|)$$

or

$$\ln(\Delta KE_t / \Delta KE_{t-1}) = a + b \cdot \ln(|\Delta ED_t|/|\Delta ED_{t-1}|) + c \cdot \ln(|\Delta S_t|/|\Delta S_{t-1}|)$$

(28)

Table 6  Summary of Regression for the Equation of Work and Kinetic Energy
(After taking the first-order differences for variables in Eq.(28))

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$\hat{a}$</th>
<th>$\hat{b}$</th>
<th>$\hat{c}$</th>
<th>F</th>
<th>Adj-R$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Whole Period</td>
<td>4.473622 (98.172)***</td>
<td>0.021125 (6.0165)***</td>
<td>0.032679 (10.338)***</td>
<td>7166.22</td>
<td>0.901563</td>
</tr>
<tr>
<td>Subperiods</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90/01/04-90/10/01</td>
<td>4.811969 (33.706)***</td>
<td>-0.022964 (-1.821)</td>
<td>0.017435 (1.417)</td>
<td>392.11</td>
<td>0.848192</td>
</tr>
<tr>
<td>90/10/02-97/07/31</td>
<td>4.389292 (97.821)***</td>
<td>0.027071 (6.954)***</td>
<td>0.035233 (10.404)***</td>
<td>5161.62</td>
<td>0.887830</td>
</tr>
<tr>
<td>97/08/01-98/03/31</td>
<td>4.986635 (79.903)***</td>
<td>0.017192 (1.607)</td>
<td>0.013432 (1.043)</td>
<td>102.64</td>
<td>0.628815</td>
</tr>
</tbody>
</table>

Note: 1. Figures in parentheses ( ) are t-values
2. * significant at 0.05, ** significant at 0.01, *** significant at 0.005 or better
3. Before taking the first-order difference, $\bar{R}^2$ are 0.1649, 0.0229, 0.0109, and 0.0016, respectively, for the four periods.

5. Conclusions

This paper applies the kinematic and kinetic theories of physics to develop two important behavior equations for the stock prices: the equation of motion and the equation of work and kinetic energy. Empirical evidence shows that these two equations provide a powerful and good description of the behavior of stock prices. The contributions of this paper are to formally theorize some language (e.g., energy of stock market, price-volume relation, etc.) that is often heard from the practitioners of stock markets, and to further understand the behavior of stock prices.

Empirical evidence regarding the equation of motion shows that an unit change in excess demand (units of buy orders minus units of sale orders) will cause the TSE index to change about 0.00034 points with the same direction of the change in excess demand. If the equation is used for prediction, we find that the longer the estimation period, the more accurate for the prediction. Using OLS method, the 4-year estimation period gives the best prediction results; however, using GARCH method, the 2-year estimation period gives the best prediction results. Interestingly, all average prediction errors are, in general, small and not statistically significant.

Empirical evidence regarding the equation of work and kinetic energy shows that change in kinetic energy (i.e., percentage change in trading values) is equal the work done (i.e., the absolute value of percentage change in excess demand times the absolute value of percentage change in stock prices). This principle provides a rather high explanatory power for the Taiwan stock market.

Although these two principles for the stock markets provide a very good description of stock price behavior for the historical data of the Taiwan stock market, this does not imply that the development of the two equations violates the hypothesis of market efficiency since we assume that changes in excess demand are deterministic. However, small prediction errors for the equation of motion imply that stock prices are predictable for some persons who own the information of buy and sale orders. To avoid this opportunity and to provide a fair market, the time interval between two successive matches of transactions should be shortened. The shorter, the better. Also, the development of these two principles does not reject the application of the Brownian motion to the dynamic of stock prices. Indeed, if changes in excess demand are random (because the arrival of information is random), then the
dynamic of stock prices must also be stochastic. Ex post data provides good fits of the theories, this does not mean stock prices are predictable. Future research can focus on the applications of the equation of work and kinetic energy and on the empirical evidence of the principle of impulse and momentum for stock markets. If data for individual stocks are available, empirical studies can focus on the implications of the equations. For example, other things being equal, the larger the “mass” of a stock, the smaller the change in stock price. Finally, future applications of these models can be explored in stock markets and futures markets to enhance profits for investors.

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References