Capacity Control and Distribution Problem for Manufactures in Supply Chain Networks

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Abstract

Most manufacturing firms have focused on managing efficiently their supply chains that purchasing raw materials, producing final products, and supplying them to retailers. Since a supply chain network is composed of several stages and components, a little variation of retail sales may result in significant changes for each component on supply chains. In this view, a manufacturer is expected both to synchronize its products with the retailer’s demand and to coordinate the ordering of raw materials with production processes so that both raw materials and final goods inventories are reduced. In general, the market for final goods can be grouped into different segments, and suppliers can sell the same goods or services to different segments for different price and supply policies to maximize their total revenues. That is the basic concept of RM (Revenue Management) techniques. The success of airline RM has been widely reported, and stimulated development of RM systems for other transportation and service sectors such as hotels, cruise lines, rental cars, retail etc. (McGill and Van Ryzin, 1999; Feng and Xiao, 2006). This paper addresses an integration of SCM(Supply Chain Management) and RM problems in manufacturing systems, specifically, the simultaneous determination of procurement of raw materials, production plan and supply policy for each customer in the circumstance of demand uncertainty. We focus on modeling our problem as a stochastic dynamic programming model. Applying RM techniques, we will develop an optimization model to solve our comprehensive problem encountered in manufacturing, and some computational results with randomly generated problems are reported.

Keywords: Supply chain network; Integration model on SCM-RM; Revenue management; Stochastic dynamic programming model

1. Introduction

Most manufacturing firms have focused on managing efficiently their supply chains that purchasing raw materials, producing final products, and supplying them to retailers. Since a supply chain network is composed of several stages and components, a little variation of retail sales may result in significant changes for each component on supply chains. For example, the increase of retail sales will make a new order for final goods to manufacturers, and they may operate their facilities to meet the order, and in turn, they may also order additional raw materials to out-side resource providers. In this view, a manufacturer is expected both to synchronize its products with the retailer’s demand and to coordinate the ordering of raw materials with production processes so that both raw materials and final goods inventories are reduced. However, since the demand of final goods is uncertain and there are complex trade-offs among managing costs for components in supply chains, it is very hard to optimize simultaneously the management of entire supply chain network: the procurement of raw materials, the production of final products and the distribution of final products to retailers simultaneously.

Supply chain management (SCM) problem has been carried out in many literatures. Since optimizing SCM as a whole is too complex to solve efficiently in a single framework, it has been dealt with various forms in two sub-problems: dynamic pricing-production and procurement-production. A dynamic pricing and production problem addresses the simultaneous determination of pricing and inventory replenishment strategies in the environment of demand uncertainty. While the procurement-production problem is to determine the ordering policy of raw material and the production strategies given the uncertain demand. The major objective of these problems is to maximize the profit from minimizing the inventory level via the optimal control of the price and the production, or the price and the procurement. Revenue management techniques have been applied to solve these sub-problems.

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In addition to the optimal control of pricing and production, it is also important to optimize the supply policy for retailers to maximize the profit of the manufacturers. The optimal supply policy for each retailer's uncertain request may lead to optimize the production and the procurement which result in the minimization of inventories for raw material and final products. In this paper, we focus on optimizing the procurement, the production and the supply policies for manufacturers by controlling supply strategies to the retailers to maximize their profits.

In general, the market for final goods can be grouped into different segments, and suppliers can sell the same goods or services to different segments for different price and supply policies to maximize their total revenues. That is the basic concept of RM techniques. The success of airline RM has been widely reported, and stimulated development of RM systems for other transportation and service sectors such as hotels, cruise lines, rental cars, retail etc. (McGill and Van Ryzin, 1999, Feng and Xiao, 2006). In this paper, applying RM techniques, we will develop an optimization model to solve our comprehensive problem encountered in manufacturing.

Among various RM techniques, price and capacity controls have played key roles. In the airline industry, for example, fierce competition forces airlines to offer comparable fares in each segmented market. While price discrimination remains a primary option, competition often hinders suppliers from using the strategy liberally. In contrast, capacity allocation among different customer classes is solely under suppliers' control (Feng and Xiao, 2006). With the similar fare structure, airlines protect their revenues by accepting or rejecting booking requests from each customer class without going to price war. It is the reason why there is a large literature in the area of capacity allocation problems.

Over the last three decades, there are many publications describing the practice and theory of RM in airlines. Most of them are for capacity allocation problems. An useful approach to the capacity allocation problem was suggested in 1972 by Littlewood. He studied a stochastic two-price, single-leg airline revenue management model and proposed a marginal seat revenue principle (Weatherford and Bodily, 1992). The principle suggests that booking requests for the lower fare be declined if the seat could be sold later at the higher fare. This simple rule for seat control marked the beginning of RM in airlines. A continuous version of Littlewood's rule was derived by Bhatia and Parekh(1973), Richter(1982) and Belobaba(1987). Richter (1982) developed simple decision rules to determine optimal booking limits in a nested fare inventory system. Belobaba(1987) proposed a generalized version of Littlewood's equation to obtain allocations for more than two fare classes. Belobaba’s method, called the EMSR (Expected Marginal Seat Revenue) model, results in allocating a fixed quantity of seats to each fare class. The multiple-class problem was further studied by Curry (1990), Wollmer (1992), Brumelle and McGill (1993), and Robinson (1995). These studies showed that Belobaba's heuristic is sub-optimal.

McGill and Van Ryzin(1999) considered the airline revenue management problem without cancellations and no-shows and they assumed that lower fare classes book strictly before higher ones. They proposed a simple adaptive approach which solves Littlewood's equation using a Robbins-Monro stochastic approximation scheme. Subramaniam et al.(1999) modeled the seat allocation problem with cancellations and no-shows as a Markov Decision Process. They divided the booking time horizon into a finite number of decision periods, where each period has specified probabilities for events such as demand arrivals, cancellations, and null event. For this set up, they develop backward recursion Dynamic Programming equations. Other studies on airline revenue management problems can be found in the literature review in McGill and Van Ryzin(1999).

While revenue management techniques have been considered primarily as a tool of service operations, they have considerable potential for manufacturing operations. In a supply chain, manufacturers have faced the uncertain demand for final products, the undesirable inventory for raw materials and final products. To meet the demand at fixed intervals, manufacturers may carry large inventories of raw materials and final goods at their hands. This results in the increase of operating costs for manufacturers. For increasing the profits, efficient managing tools are needed to reduce the inventories. As in airline industry, RM in manufacturing may result in the increase of its revenue from differential pricing and dynamic capacity reallocation policies by segmented markets.

There is some literature on dynamic pricing and capacity allocation for managing supply chains with uncertain demand in manufacturing industry. They have been carried out under different assumptions. Studies of pricing strategies in revenue management motivated researches on production-pricing problems in manufacturing where finite periods were considered. Production-distribution and procurement-production problems in manufacturing could be considered as capacity allocation in revenue management problems.

Gallego and Van Ryzin (1994) considered a production-pricing problem and formulated it into a comprehensive pricing model. They applied the dynamic pricing strategy to revenue management problems and derived the optimal policy in closed-form when de-
mand functions are exponential. For general demand functions, they analyzed a deterministic version of the model and obtained an upper bound for the optimal revenue.

Feng and Gallego (1995) studied a two-price, continuous-time revenue management model with general demand functions and developed an optimal pricing policy in closed form. Feng and Xiao (1999, 2000) generalized the two-price model to consider risk preference and multiple prices. Feng and Xiao (2000) further proposed a continuous-time, dynamic pricing model which allows reversible price changes. They showed that the price reversal policy leads to an improvement of revenue and profit. Maglaras and Meissner (2003) focused on the dynamic pricing strategy for multiple products. Incorporating the joint capacity constraints and the cross-price effects, they developed an efficient way of addressing multi-product revenue management problems.

Federgruen and Heching (1999) addressed the simultaneous determination of pricing and inventory replenishment strategies in the face of demand uncertainty. Since demands are independent but their distributions depend on the item’s price, the price charged in any given period can be specified dynamically as a function of the state of the system. They addressed both finite and infinite horizon models, with the objective of maximizing total expected discounted profit or its time average value, assuming that prices can either be adjusted arbitrarily (upward or downward) or that they can only be decreased. Feng and Xiao (2006) proposed a continuous-time model that integrates pricing and inventory control decisions for perishable products. They assumed that the supplier serves multiple customer classes with the same products. Demand is a non-homogeneous Poisson process whose intensity is a function of price and time. To maximize the expected revenues under time and capacity constraints, they developed a model to decide the customer class and the corresponding price simultaneously at a given time and inventory level.

Procuring raw materials is one of important processes in supply chain networks. Sarker and Parija (1996) developed an ordering policy for raw materials to meet the demands of a production facility which supplies final products to markets. In their model, an optimal multi-order policy for procurement of raw materials for a single manufacturing batch was developed to minimize the total cost, and an integer approximation was adopted to refine the optimal solution. Polatoglu and Sahin (2000) studied a periodic-review inventory model where, in addition to the procurement quantity, price is also a decision variable. They developed a model where demand in each period is a random variable having a price dependent probability distribution, with the expected demand decreasing in price. The model included price limits and fixed ordering costs in addition to unit procurement holding and shortage costs. They studied the optimal policies which jointly maximize the discounted expected profit over a finite planning horizon, and characterized the form of the optimal procurement policy under a general price-demand relationship and give a sufficient condition for it.

Another important process in supply chain networks is a production-distribution problem. Ganeshan (1999) considered a two-level production/distribution system that operates via many identical retails through the lower level. Their model was a synthesis of three components: (i) the inventory analysis at the retailers, (ii) the demand process at the warehouse, and (iii) the inventory analysis at the warehouse. They developed a near-optimal inventory policy for a production/distribution network with multiple suppliers replenishing a central warehouse, which in turn distributes to a large number of retailers. Chen and Samroengraja (2000) considered a distribution system with one warehouse and N retailers. Random demands occurred at the retailers only, and excess demands are completely backlogged. The retailers replenished their inventories from the warehouse, which in turn orders from an outside supplier assumed to have unlimited stock. They developed an approximate model to determine near-optimal control parameters for allocation policies.

All of these researches have focused on the revenue management problem itself or introducing RM concepts in the some part of SCM process. For manufacturers, it needs to control their whole supply chain networks—procurement, production and supplying—-for maximizing the revenue from avoiding excess inventories on raw materials and final products under uncertain demand. For this, it is necessary to build an integration of SCM and RM for manufacturers. Unfortunately, we couldn’t find any research on the integration of SCM and RM with uncertain demand. This paper addresses an integration of SCM and RM problems in manufacturing systems - specifically, the simultaneous determination of procurement of raw materials, production plan and supply policy for each customer in the circumstance of demand uncertainty.

The rest of this paper is organized as follows. In section 2, we briefly describe our problem and formulate it as a stochastic dynamic programming model. Extensive computational results with randomly generated problems are reported in section 3, and some concluding remarks and extensions of our research are given in the last section.
2. Revenue Management Model in Supply Chain-Networks

Consider a manufacturer operating a single raw material and a single product. We assume that the manufacturer purchases raw materials from an outside provider and processes them to deliver final goods or services to multiple retailers. The raw material is non-perishable, and it is supplied instantaneously to the manufacturing facility. The demand of each retailer is uncertain, but its distribution is known. Given a production capacity and a finite planning horizon, manufacturers should produce final goods and keep them to meet the demand requests. When the manufacturer serves a retailer, a pre-determined price is applied and the amount of supply is selected dynamically from the supplying policies by retailers at each time. We assume that there is no ‘buy-up’ or ‘buy-down’ among different retailers.

In the whole supply chain, we should coordinate the procurement, production and distribution strategies dynamically to maximize the total expected revenue from avoiding excess inventories for raw materials and final products under uncertain demand. From a manufacturer’s view, a supply chain can be depicted as Figure 1. A manufacturer obtains raw materials from an external provider, and processes them with the existing inventory to make final products. The final product with its existing inventory can be supplied to several markets.

In this supply chain network, manufacturers can divide customer groups by contract conditions and/or the scale of demand, and provide different prices for different customer groups. A manufacturer should decide the time and the amount of raw materials to be purchased and final goods to be produced, as well as the supply for each market to meet its uncertain demand request. Due to the uncertainty of demand requests, it is a very complex problem to find an optimal procurement-production-supply policy for maximizing the total profit. When an orders arrive from a low price market, manufacturers have to decide whether to reject, or to fulfill all or part of the order. If the order is accepted at the low price, the manufacturer may loss the opportunity to sell the final products to the high price market. Alternatively the manufacturer could produce final goods with additional costs. If the future demand from the high price market will be not enough to consume all capacity, it is better to sell the final product at the low price market.

We assume that the price for each segmented market is different with others due to trade conditions and company’s policy. Once the demand request from a market has arrived, the manufacture can accept all or some part of it. Under the current resources, the inventory for products and production capability with raw materials, the manufacturer may reject some part of the demand request to remain the part of current resources. The remaining resources may be supplied to more profitable markets which may occur in the future for higher profit. This can be found in the manufacturing industries having large set-up costs for productions or procurements such as automotive, steel and semiconductor industries. Therefore, it is need to have an efficient control scheme for manufactures on the demand request of each segmented market. For the demand request of each market, manufactures should decide the supply policy for it under the current status for the market price of products and the resource availability. Manufactures have to have an efficient criterion to determine the acceptance for all or a part of demand requests. We call it as the cap for each market. Once the demand request is less than or equal to the cap for the market, it is supplied fully from the manufacturer. However, if the demand request is more than the cap, the maximum supply is up to the cap.

We assume that the demand of each market is uncertain, and independent to the production and procurement policies. In this paper, we will develop a discrete-time dynamic programming model.

3. Discrete-time Dynamic Programming Model

We assume the whole market can be classified into
$M$ segmented markets, and the demand of each segmented market is random and independent. Let $T$ be the number of decision periods or stages. At each time or stage, we assume that only one of the two events occurs: (1) an arrival of a demand request for market $j = 1,2,...,M$ or, (2) a null event. Let $p_j^t$ and $q_0^t$ denote the probabilities for a demand request of $j$ and the null event occurring at time $t$ respectively. With our assumption, we have

$$\sum_j p_j^t + q_0^t = 1 \text{ for } t=1,2,...,T.$$  \hfill (1)

If the event occurring at time $t$ is the arrival of demand request, the manufacturer should decide the amount of products being supplied based on the price for the market $j$ and the current state of inventories for products and raw materials. Furthermore, the manufacturer may determine the production plan for the final goods and adjust the procurement schedule for raw materials. Even though the event is null, the production and the procurement plans may be adjusted for supporting future demands.

In this paper, we use the following notations and variables:

- $r_j^t$: product price for a market $j$ at $t$
- $c_p^t$: unit cost for producing a product at $t$
- $K_j^t$: fixed cost for a production at $t$
- $c_f^t$: unit cost for purchasing a raw material at $t$
- $h_f^t$: unit cost for inventory of a raw material at $t$
- $h_p^t$: unit cost for inventory of a product at $t$
- $X_j$: set of alternatives for amount of procurement,
- $Z_j$: set of alternatives for amount of production,
- $d_j^t$: demand for market $j$ at $t$, random variable,
- $Q_j$: production capacity at $t$
- $I_j^t$: amount of inventory for products at the end of $t$
- $I_j^t$: amount of inventory for products at the end of $t$
- $w$: conversion ratio from a resource to a product.
- $s_j^t$: amount of supply for market $j$ at $t$
- $z_j^t$: amount of supply threshold for market $j$ at $t$
- $y_j$: number of production at $t$, $y_j \in Y$
- $x_j$: amount of raw material purchase at $t$, $x_j \in X$

Assume the inventories of raw materials and final products at the beginning of time $t$ are $I_x^{t-1}$ and $I_y^{t-1}$ respectively. Once the procurement $x_j$, the production $y_j$ and the supply $s_j^t$ for demand $j$ at time $t$ are given, let $H_j^t(s_j^t | x_j, y_j, I_x^{t-1}, I_y^{t-1})$ be the expected profit from $t$ to the end of planning period $T$ for the demand $j$.

$$H_j^t(s_j^t | x_j, y_j, I_x^{t-1}, I_y^{t-1})$$  \hfill (2)

\[
\begin{align*}
&= \mathbb{E}[r_j^t \text{Min}(d_j^t, z_j^t) - h_f^t(I_x^{t-1} + x_j - wy_j) - h_p^t(I_y^{t-1} + y_j - \text{Min}(d_j^t, z_j^t)) - c_p^t x_j + \Pi_t^s(I_x^{t-1} + x_j - wy_j, I_y^{t-1} + y_j - \text{Min}(d_j^t, z_j^t)) - P_t(y_j),]
&\text{ where } I_x^{t-1} + x_j - wy_j \geq 0, I_y^{t-1} + y_j - z_j^t \geq 0, z_j^t \in Z_j,
\end{align*}
\]  \hfill (3)

The optimal supply for the demand $j$ can be obtained from maximizing $H_j^t(s_j^t | x_j, y_j, I_x^{t-1}, I_y^{t-1})$ under the inventory constrains. Given $(x_j, y_j, I_x^{t-1}, I_y^{t-1})$ at time $t$, let $G_j^t(x_j, y_j, I_x^{t-1}, I_y^{t-1})$ be the maximum value of $H_j^t(s_j^t | x_j, y_j, I_x^{t-1}, I_y^{t-1})$ for all $z_j^t$.

$$G_j^t(x_j, y_j, I_x^{t-1}, I_y^{t-1}) = \max \{ H_j^t(z_j^t | x_j, y_j, I_x^{t-1}, I_y^{t-1}) | I_x^{t-1} + x_j + wy_j \geq z_j^t, z_j^t \in Z_j \}$$  \hfill (4)

On the other hand, when the null event is occurred at time $t$, the total expected value from $t$ to the end of planning period $T$ can be defined as $G_j^t(x_j, y_j, I_x^{t-1}, I_y^{t-1})$:
\begin{align*}
G_q^t(x_t, y_t | I_x^{t-1}, I_y^{t-1}) &= -c_r^t x_t - h_r^t(I_x^{t-1} + x_t - w y_t) - h_p^t(I_y^{t-1} + y_t) \\
&+ \Pi^*_t(I_x^{t-1} + x_t - w y_t, I_y^{t-1} + y_t) - P_t(y_t), \\
\text{where } &I_x^{t-1} + x_t - w y_t \geq 0, \ I_y^{t-1} + y_t \geq 0.
\end{align*}

With our threshold policy for supply and given the inventories for the raw materials and the final goods at time \( t \), the optimal values for the procurement, the production and the supply can be determined by maximizing the total expected profit from \( t \) to the end of planning period \( T \). Let \( \Pi_t^*(I_x^{t-1}, I_y^{t-1}) \) be the total expected profit from \( t \) to the end of planning period \( T \), when the inventories of raw materials and products at the beginning of time \( t \) are given as \( I_x^{t-1} \) and \( I_y^{t-1} \) respectively. Since the probabilities for a demand \( j \) and a null event at time \( t \) are given as \( p_j^t \) and \( q_0^t \) respectively, \( \Pi_t^*(I_x^{t-1}, I_y^{t-1}) \) is represented as the following equation (6) for maximizing the total expected profit from \( t \) to \( T \):

\begin{equation}
\Pi^*_t(I_x^{t-1}, I_y^{t-1}) = \max \{ \sum_j p_j^t G_j^t(x_t, y_t | I_x^{t-1}, I_y^{t-1}) + q^t G_q^t(x_t, y_t | I_x^{t-1}, I_y^{t-1}) \ |
I_x^{t-1} + x_t - w y_t \geq 0, \ y_t \leq Q_t, \ x_t \in X, \ y_t \in Y \}
\end{equation}

\( \Pi_t^*(I_x^{t-1}, I_y^{t-1}) \) can be obtained by maximizing the expected value of the profit for all event at time \( t \) with constraints such as production capacity, inventory amount for raw materials and final goods. Obviously, \( \Pi_{T-1}^*(I_x^T, I_y^T) = \Phi_{T-1}(I_x^T, I_y^T) \), where \( \Phi_{j}^r(\cdot) \) is a concave penalty function, and \( \bar{y}_j^r, \bar{x}_j^r, \bar{z}_j^r \) denote the optimal amounts for the procurement, the production and the supply for maximizing \( \Pi_t^*(I_x^{t-1}, I_y^{t-1}) \) at time \( t \) respectively. Equation (6) is a stochastic dynamic programming with the state space \( (I_x^{t-1}, I_y^{t-1}) \) and \( T \) stages. Even though we can solve our problem by applying equation (6), it requires huge number of state spaces at each stage which result in the increase of computation burden. To reduce the state spaces at each stage, we can consider an approximation technique exploiting the following property of \( \Pi_t^*(I_x^{t-1}, I_y^{t-1}) \).

**[Theorem 1]** If \( \Pi_{T-1}^*(I_x^T, I_y^T) \) is a concave, piecewise linear function than so is \( \Pi_t^*(I_x^{t-1}, I_y^{t-1}) \), provided all \( d_j^r \)'s have discrete probability distributions.

\begin{align*}
\max \{ r_j^t, \min(d_j^t, z_j^t) \} - c_r^t x_t - h_r^t(I_x^{t-1} + x_t - w y_t) - h_p^t(I_y^{t-1} + y_t) - \min(d_j^t, z_j^t) \\
+ \Pi_{T-1}^*(I_x^T, I_y^T) - P_t(y_T) [x_T \leq Q_T, \ x_T \in X, \ y_T \in Y, z_j^T \in Z_j] \}, \ j T-1 \) be the total expected profit for \( \Pi_{T-1}^*(I_x^T, I_y^T) \) only calculating on some fixed points for \( I_x^{T-1} \) and \( I_y^{T-1} \) instead of considering full spaces of states. The more points are considered, the tighter lower bound is obtained and the more computation time is required.

Since (6) is a backward DP formulation, it may give us information to find an optimal decision at any time \( t \) maximizing the expected value for remaining planning period. Once the optimal expected values \( \{ \Pi_t^*(I_x^{t-1}, I_y^{t-1}) \} \) for all \( t \), \( I_x^{t-1} \) and \( I_y^{t-1} \) are obtained by (6), we can use them to control each event occurring at time \( t \). When the demand \( j \) or the null event is arrived at time \( t \), the optimal policies for the procurement, the production and the supply should be determined by the following equations to maximize the total expected values from \( t \) to \( T \):

\begin{align*}
\max \{ r_j^t, \min(d_j^t, z_j^t) \} - c_r^t x_t - h_r^t(I_x^{t-1} + x_t - w y_t) - h_p^t(I_y^{t-1} + y_t) - \min(d_j^t, z_j^t) \\
+ \Pi_{t+1}^*(I_x^{t+1}, I_y^{t+1}) - P_t(y_t) [x_T \leq Q_T, \ x_T \in X, \ y_T \in Y, z_j^T \in Z_j] \}, \ j = 1, 2, \ldots M, \hspace{1cm} (7)
\end{align*}

\begin{align*}
\max \{ -c_r^t x_t - h_r^t(I_x^{t-1} + x_t - w y_t) - h_p^t(I_y^{t-1} + y_t) + \Pi_{T-1}^*(I_x^T, I_y^T) - P_t(y_T) \} \rbrace, \\
I_x^{t-1} \geq 0, I_y^{t-1} \geq 0, \ x_T \leq Q_T, \ x_T \in X, \ y_T \in Y \}, \text{ for the null event.} \hspace{1cm} (8)
\end{align*}

**4. Computational Experiments**

This section presents the simulation results to demonstrate the validity of DP formulation (6) for maximizing the total expected profit in supply chain networks. Since (6) requires lots of state spaces in the computation processes to find the optimal solution, it may take the extra long time for computation. To reduce the computational burden, we will try to find good feasible solutions providing the lower bound for the optimal value by the approximation based on the Theorem 1. We assume the salvage values for the remaining products and raw materials at the end of planning period are equal to zero \( \Pi_T^*(I_x^T, I_y^T) = 0 \). Since this satisfies the necessary condition for the Theorem 1, we can apply the approxi-
mination method for calculating $\Pi^t(I_x^t, I_y^t)$ and may consider the total expected values on a limited number of points on $(I_x^t, I_y^t)$ to obtain the lower bound for the total expected profit at time $t$. Once the total expected values for two points on $(I_x^t, I_y^t)$ are obtained from (6), the value for any point between these two points can be calculated by the interpolation, and it gives a lower bound for the objective value.

**Table 1. Parameters for Simulation**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Planning Period $T$</td>
<td>40</td>
</tr>
<tr>
<td>Number of Retailers $M$</td>
<td>10</td>
</tr>
<tr>
<td>Costs $K_t = 1000/5000, c_t^f = 20, c_t^r = 30$</td>
<td></td>
</tr>
<tr>
<td>$h_t^r = 0.1/0.4, h_t^p = 0.2/0.6$</td>
<td></td>
</tr>
<tr>
<td>Maximum Capacity $Q^*_t$</td>
<td>$5^*\text{Max Demand}$</td>
</tr>
<tr>
<td>Number of Demand $N_s$</td>
<td>15</td>
</tr>
<tr>
<td>Number of supply thresholds $N_r$</td>
<td>15</td>
</tr>
<tr>
<td>Set of supply thresholds ${\text{Max Demand} \times ( i/N_s + 0.5), i = 1, 2, ..., N_s}$</td>
<td></td>
</tr>
<tr>
<td>Number of alternatives for procurements $N_p$</td>
<td>15</td>
</tr>
<tr>
<td>Set of alternatives for procurements ${3 \times Q_t \times i / N_s, i = 1, 2, ..., N_s}$</td>
<td></td>
</tr>
<tr>
<td>Number of alternatives for production $N_p$</td>
<td>15</td>
</tr>
<tr>
<td>Set of alternatives for production ${Q_t \times i / N_p, i = 1, 2, ..., N_p}$</td>
<td></td>
</tr>
</tbody>
</table>

For simulation study, the approximation of the recursive equation (6) was coded in C and tested on randomly generated problems with given parameters and assumptions. All tests were carried on a PC (Pentium 4, 1.8GHz). For exposition brevity, we assume all parameters are independent to the time $t$. In the simulation, we assume that the demand of each retailer is discrete, with possible values randomly generated from the interval $(\alpha_1, \alpha_2) = (10, 80)$. The probability for each demand is also generated randomly, but the sum of probabilities is equal to 1. Since the price for final products may be differentiated by retailers, the price offered to each retailer is generated in the interval $(b_1, b_2) = (100, 200)$ randomly. Furthermore, the probabilities $p_j, q_0$ of any event occurring is generated randomly. The other parameters assumed in the simulation are given in Table 1.

Table 2 shows the computational results for test problems. The objective values in Table 2 are not the optimal values, but the lower bounds providing feasible solutions. As expected, we can see the more intervals for computing $\Pi^t(I_x^t, I_y^t)$ in the approximation, the higher profit is obtained and the more computation time is required. In Table 2, when the number of intervals for the inventories of raw materials and final products are both 50, we can obtain more profitable solutions, but the computation times are required more than 2 hours in PC.

**Table 2. Computational Results**

<table>
<thead>
<tr>
<th>Test Set</th>
<th>$K_t$</th>
<th>$I_x^t$</th>
<th>$I_y^t$</th>
<th>Objective Values</th>
<th>Average Supply</th>
<th>Computation Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-1</td>
<td>1800</td>
<td>0.4</td>
<td>0.6</td>
<td>(10, 30)</td>
<td>34.085</td>
<td>83.2</td>
</tr>
<tr>
<td>T-2</td>
<td>2000</td>
<td>0.5</td>
<td>0.7</td>
<td>(30, 20)</td>
<td>57.296</td>
<td>70.3</td>
</tr>
<tr>
<td>T-3</td>
<td>2500</td>
<td>0.4</td>
<td>0.5</td>
<td>(30, 30)</td>
<td>60.743</td>
<td>73.7</td>
</tr>
<tr>
<td>T-4</td>
<td>1500</td>
<td>0.3</td>
<td>0.2</td>
<td>(40, 40)</td>
<td>65.524</td>
<td>81.3</td>
</tr>
<tr>
<td>T-5</td>
<td>2000</td>
<td>0.4</td>
<td>0.6</td>
<td>(30, 20)</td>
<td>60.045</td>
<td>92.0</td>
</tr>
<tr>
<td>T-6</td>
<td>1500</td>
<td>0.4</td>
<td>0.5</td>
<td>(30, 30)</td>
<td>64.156</td>
<td>96.3</td>
</tr>
</tbody>
</table>

For exposition brevity, we assume all parameters are independent to the time $t$. In the simulation, we assume that the demand of each retailer is discrete, with possible values randomly generated from the interval $(\alpha_1, \alpha_2) = (10, 80)$. The probability for each demand is also generated randomly, but the sum of probabilities is equal to 1. Since the price for final products may be differentiated by retailers, the price offered to each retailer is generated in the interval $(b_1, b_2) = (100, 200)$ randomly. Furthermore, the probabilities $p_j, q_0$ of any event occurring is generated randomly. The other parameters assumed in the simulation are given in Table 1.

Table 2 shows the computational results for test problems. The objective values in Table 2 are not the optimal values, but the lower bounds providing feasible solutions. As expected, we can see the more intervals for computing $\Pi^t(I_x^t, I_y^t)$ in the approximation, the higher profit is obtained and the more computation time is required. In Table 2, when the number of intervals for the inventories of raw materials and final products are both 50, we can obtain more profitable solutions, but the computation times are required more than 2 hours in PC.

The increase of the fixed cost for the production may affect little in the computation time, but make a little bit change in the amount of supply for each market. Usually, manufacturers tend to supply their products to the high profit demand more. Once the fixed cost for the production is increased, the high profit demand is reduced and thereby we can see the average supply rates are decreased a little bit in Table 3.

Since the number of states increases rapidly with the number of time periods $T$ in the DP formulation (4), the computation time is also increasing with $T$. However, in this study we considered the approximation method for finding $\Pi^t(I_x^t, I_y^t)$ in which the number of states is limited to the certain number and for which the objective provides the lower bound. With the approximation, the computation time increases linearly with $T$. Owing to the consideration of a limited number of states in the approximation, the objective values are decreased as the increase of $T$. While, we can see the lower bounds are increased with the number of states considered in the approximation. Figure 2 illustrates the results.

To compare the performance of our model, we consider an ad-hoc supply policy: if the demand request is
greater than or equal to the current available inventory, the supply amount is up to the inventory levels. However, once the demand request is less than the current inventory, the supply amount is the same that of the requests. For the supply policy, the actual supply is \( M_{j} \cdot I_{y} + t \).

Table 3 shows the computation results for both supply policies. As expected, the threshold supply policy is more profitable than the ad-hoc policy for all test sets. However, it requires more computation time to obtain the solution.

For all test sets, the computation times of the threshold policy is more than 10 times that of the ad-hoc policy.

5. Conclusions

In this paper, we focused on building the procurement, the production and the supply policies in supply chain networks to maximize the total expected profit. In particular, we considered an adaptive supply policy for manufacturers in which the amount of supply is varied and the price is differentiated among retailers. Since the demand requests are uncertain (stochastic), the fixed supply policy which the amount of supply is determined a priori is not suitable to manage the uncertain demand. To demonstrate the performance of the expected profit (6) for our supply policy, our results were compared with those for the ad-hoc supply policy considering the average demand of retailers.

From the computation experiments, we can see that the recursive definition of expected profit (6) can be applied to get the important decisions in the supply chain networks efficiently. Using a dynamic programming algorithm based on (6) and an appropriate approximation, we can obtain good decisions for the procurement of raw materials, the production plan of final products and the supply policy for market demands considering their differentiated prices. In particular, the threshold supply policy in supply chain networks for manufacturers will be useful to contribute the increase of the expected profit when there are different market prices.

References


Feng, Y. and Xiao, B. (1999). Optimal overbooking policy for a single-leg flight with time-dependent demands. Working paper, Na-


