The Effect of Signals in Informed Traders’ Duopoly

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Abstract

This paper analyzes the effect of signals based on Kyle [8] and Kyle and Wang [10]. Kyle and Wang [10] show that there exists a unique linear equilibrium in informed traders' duopoly. We focus on the difference of private signals and consider that the difference is caused by the difference of the informed traders' prior beliefs. When two informed traders' beliefs are heterogeneous, two models are provided. First we analyze the model based on Kyle and Wang [10]. We show the existence of a unique linear equilibrium and the irrational trader's expected profit can be larger than the rational trader's. Next we revise this model to one that the difference of signals affects the informed traders' prediction of the pricing rule. Then, one informed trader can perfectly react the opponent trader's trading strategy, each other. These results are caused by their correct understanding of the opponent signal.

Keywords: Kyle model; Informed trader; Private signal; Prior belief; Pricing rule

1. Introduction

In this paper, we consider the security market with two kinds of informed traders, liquidity traders and market makers. Each informed trader can acquire the information about the ex post liquidation value of the risky asset. It is assumed that they observe the information as signals and determine their own orders. Liquidity traders have no information and trade randomly. Market makers set trading price in order to clear the market after observing the market order imbalance.

We analyze this market model as informed traders' duopoly based on Kyle [8] and Kyle and Wang [10]. Kyle [8] analyzes the case of single insider. He shows that there exists a unique linear equilibrium and liquidity trading provides camouflage which conceals informed trading and, as a result, the informed trading is swamped by liquidity trading. And he shows that the market liquidity is determined in the model.

Kyle and Wang [10] consider heterogeneous prior beliefs in their study of the survival of overconfident traders in a duopoly context. We use the
same method in this paper as Kyle and Wang [10], and we show that there exists a unique linear equilibrium. First, we consider the case that two informed traders have the same private signals, and next consider the case of different private signals.

The difference of private signals that two informed traders observe is caused by the heterogeneous prior beliefs. The traders with irrational prior belief misperceive the quality of their signals.

Irrational prior belief, as a well-known cognitive error, may be due to an “anchoring and adjustment” process described in Solvic and Lichtenstein (1971) and Tversky and Kahneman [14]. Such a bias has been noticed in a large variety of professional including clinical psychology, law, management and economics. Odean (1995) contains a good summary of psychology literature on irrational belief.

There are many papers that analyze the informed trading with private information (signals). In rational expectations context, there are Grossman [3], Grossman and Stiglitz [4], Verrecchia [15] and Hirshleifer and Luo [5], etc. And a lot of papers about the survival of overconfident traders are published by authors (for example, Kyle and Wang [10], Benos [1] and Hirshleifer and Luo [5], etc.).

We investigate three types of models, that model 1 is the basic informed traders' duopoly with the homogeneous signal, and model 2, 3 are the model based on Kyle and Wang [10] with heterogeneous signals. The difference between model 2 and 3 is informed traders' prediction of the pricing rule set by market makers. In model 3, we consider that since two informed traders' prior beliefs are different, not only their trading intensity but the prediction of the pricing rule is different. The informed traders' actions are analyzed in each model.

This paper is structured as follows. Section 2 develops a duopoly by informed traders who have the same prior belief and the unique linear equilibrium. In equilibrium each informed trader's strategy is characterized by a trading intensity parameter. Since one trader's intensity parameter is a linear function of the other trader's intensity parameter, these intensity parameters are analogous to quantity choices in a standard Cournot duopoly model with linear response functions. In this case, the expected profits of both informed traders are positive. In section 3 and 4, model 2 and 3 described above are analyzed respectively. In section 5 we discuss on the parameters and the expected profit in equilibrium of each model. And the section 6 concludes
The main result of this paper is that if two informed traders know the difference of the prior beliefs, they can predict the decision process of the opponent trading quantities, each other, correctly. Then, they can adjust their trading strategies perfectly.

2. Informed Traders’ Duopoly

2.1 Model 1

Based on Kyle [8] and Kyle and Wang [10], we analyze a one-shot market where a single risky asset with an ex post liquidation value \( \tilde{v} \) is traded. It is assumed that there are three types of risk-neutral traders, informed traders, liquidity traders and market makers. Trader 1 and trader 2 are informed traders (insiders) and they can acquire the information about \( \tilde{v} \) as signals. After they observe the signal \( \tilde{v} = v \), they submit their market orders \( \tilde{x}_1 \) and \( \tilde{x}_2 \), respectively. Liquidity traders have no information about \( \tilde{v} \), and don’t trade strategically. Their aggregate trading quantity is exogenous random variable \( \tilde{z} \). Then market makers clear the market and set the trading price \( \tilde{p} \) efficiently, after observing the market order imbalance \( \tilde{y} = \tilde{x}_1 + \tilde{x}_2 + \tilde{z} \).

2.2 Trading Structure and Assumptions

The ex post liquidation value \( \tilde{v} \) is normally distributed with mean \( \tilde{v} \) and variance \( \sigma^2_v \). Without loss of generality, we assume \( \tilde{v} = 0 \). The quantity traded by liquidity traders, denoted \( \tilde{z} \), is normally distributed with mean zero and variance \( \sigma^2_z \). The random variable \( \tilde{z} \) is independent of any other variables.

Trading is structured in two steps as follows:

In step one, the exogenous value \( \tilde{v} \) and \( \tilde{z} \) are realized, and trader 1 and trader 2 choose their trading quantities \( \tilde{x}_1 \), \( \tilde{x}_2 \), respectively. In step two, market makers determine the market clearing price \( \tilde{p} \). When doing so, they observe the market order imbalance (net order flow) \( \tilde{y} \), not each trader’s order.

Therefore, in equilibrium the profit of informed trader \( i \) (\( i=1,2 \)) is given by
\[
\tilde{x}_i = (\tilde{v} - \tilde{p})\tilde{x}_i.
\]
2.3 Equilibrium

In this model, we have to analyze the strategies of both informed traders and market makers. In equilibrium market makers are assumed that they set the price efficiently for the given market order imbalance $\tilde{y}$. It implies that the price is the expected value of $\tilde{v}$ conditional on the order imbalance $\tilde{y}$.

Now let the pricing rule of market makers to be denoted by the function $\tilde{p} = P(\tilde{y})$. Then the equilibrium price must satisfy the following condition:

$$P(\tilde{x}_1 + \tilde{x}_2 + \tilde{z}) = E[\tilde{v} \mid \tilde{x}_1 + \tilde{x}_2 + \tilde{z}] .$$

This is the market efficiency condition. $E[...]$ expresses the expectation operator of traders. As in Kyle and Wang [10], we assume the linear pricing rule.

Let the trading strategy of informed trader $i$ to be the linear function of his signal, denoted $\tilde{x} = X_i(\tilde{v})$, $i = 1, 2$, then the trading strategy depends on not only the pricing rule but also the distribution parameter of liquidity trading. Though informed traders don’t know the real order flow of liquidity trading, they recognize the parameter of distribution, and for every pattern of realized order flow they form the expectation about how market makers set price. Furthermore, one informed trader determines his own order taking into account the other informed trader’s trading strategy. As profit $\pi_i$ depends on $X_1, X_2, P$, so we write $\tilde{x}_i = \tilde{x}_i(X_1, X_2, P)$. Given the pricing rule and the opponent strategy, informed trader $i$’s optimal trading strategy must satisfy the following condition:

$$E[\tilde{x}_i(X_1, X_2, P) \mid \tilde{v} = v] \geq E[\tilde{x}_i(X_1', X_2, P) \mid \tilde{v} = v],$$

$$E[\tilde{x}_2(X_1, X_2, P) \mid \tilde{v} = v] \geq E[\tilde{x}_2(X_1, X_2', P) \mid \tilde{v} = v].$$

This is the profit maximization condition. As each trader is risk-neutral, their trading quantities are set in order to maximize the expected profits conditional on signals.

This model has an analytically tractable equilibrium in which $X_1, X_2, P$ are simple linear functions, as following theorem.

**Theorem 1:** There exists a unique linear equilibrium in which $X_1, X_2, P$ are linear functions. The equilibrium trading strategies and the price are of
the form:

\[ X_1(\bar{v}) = \alpha_1 \bar{v} \]
\[ X_2(\bar{v}) = \alpha_2 \bar{v} \]
\[ P(\bar{y}) = \lambda \bar{y} \]

where the parameters \( \alpha_1, \alpha_2, \lambda \) are given by

\[ \alpha_1 = \alpha_2 = \frac{\phi}{2} \]
\[ \lambda = \frac{2}{3\phi} \]

and \( \phi = \sigma_z / \sigma_y \).

Proof: Since we assume the linear pricing rule, the market clearing price is denoted as \( \hat{p} = P(\bar{y}) = \lambda \bar{y} \). With the linear pricing rule and the linear conjecture of trader \( j \)'s strategy \( X_1(\bar{v}) = \alpha_1 \bar{v} \), trader \( i \)'s expected profit conditional on his signal \( \bar{v} \) is given by

\[
E[ (\bar{v} - P(\bar{y})) x_i | \bar{v} = v ] = E[ (\bar{v} - \lambda (x_i + \alpha_j \bar{v} + \bar{z})) x_i | \bar{v} = v ]
\]

\[ = (1 - \lambda \alpha_j) \nu x_i - \lambda x_i^2 \] \hfill (1)

The first order condition yields the optimal strategy for trader \( i \) as follows:

\[ \nu_i^* = \frac{1 - \lambda \alpha_j}{2 \lambda} v \]

Thus, the trading intensity parameter for trader \( i \) is given by

\[ \alpha_i = \frac{1 - \lambda \alpha_j}{2 \lambda}, (i, j = 1,2, i \neq j) \] \hfill (2)

Given \( \lambda \), parameters \( \alpha_1, \alpha_2 \) can be solved simultaneously and the solution is given by

\[ \alpha_1 = \alpha_2 = \frac{1}{3 \lambda} \] \hfill (3)

Given \( \alpha_1, \alpha_2 \), the market efficiency condition implies that
The market liquidity parameter $\lambda$ is given by

$$\lambda = \frac{\alpha_1 + \alpha_2}{\alpha_1^2 + \alpha_2^2 + \phi^2}$$

Plugging equation (3) into (4), $\alpha_1, \alpha_2, \lambda$ can be solved, and the parameters are given by

$$\alpha_1 = \alpha_2 = \frac{\phi}{2}$$

$$\lambda = \frac{2}{3\phi}$$

From $\lambda = \frac{2}{3\phi} > 0$, the second order condition in informed traders' optimization problem is satisfied. Q.E.D.

2.4 Properties of the Equilibrium

The equilibrium $X_1, X_2, P$ are determined by the exogenous parameters $\sigma^2$ and $\sigma^2_v$. We call parameter $\phi$ the noise trading ratio as in Kyle and Wang [10]. It is noted that equation (2) gives trader $i$'s trading intensity parameter $\alpha_i$ as a linear Cournot response function to trader $j$'s intensity parameter $\alpha_j$.

If informed trader $j$ increases trading intensity by one unit, the other informed trader $i$'s best response is to reduce his trading intensity by 1/2 unit, holding $\lambda$ constant. So, equation (3) is the solution of Cournot equilibrium. The liquidity parameter $\lambda$ characterizes the pricing rule, and $\lambda$ is an inverse measure of market depth. Parameters $\alpha_1, \alpha_2, \lambda$ all depend on $\phi$ and they are constant.

In equilibrium the trading quantities of informed traders are given by
\( \tilde{x}_i = \frac{\phi v}{2} \). From (1), (5), (6),

\[
E[(\tilde{v} - \tilde{\rho})\tilde{x}_i | \tilde{v} = v] = (1 - \lambda \alpha_j)vx_i - \lambda x_i^2 = \frac{1}{6} \phi v^2.
\]

Therefore, the expected profits of informed traders are as follows:

\[
E[\tilde{x}_1] = E[\tilde{x}_2] = \frac{1}{6} \sigma, \sigma.
\]

Thus, the expected profits of informed traders are positive.

3. The Case of Heterogeneous Signals

3.1 Model 2

Now we consider the case that two informed traders have heterogeneous private signals respectively. We assume that a signal is a scalar multiple of the liquidation value \( \tilde{v} \). The informed trader 1 thinks \( \tilde{k}_1 \tilde{v} \) while the informed trader 2 thinks \( \tilde{k}_2 \tilde{v} \), where \( \tilde{k}_1 \) and \( \tilde{k}_2 \) are both positive scalars. Since only the ratio of the two scalars matters in our analysis, without loss of generality, we consider that trader 1’s signal is \( \tilde{v} \) and trader 2’s signal is \( \tilde{s} = K \tilde{v} \), (\( K = k_2/k_1 \), \( K > 0 \)). Given this normalization, traders’ heterogeneous signals are captured by the belief parameter \( K \). If \( K > 1 \), the trader’s prior belief is underconfident and he tends to evaluate the variance of \( \tilde{v} \) large. Conversely, if \( 0 < K < 1 \), the trader’s prior belief is overconfident and he tends to evaluate the variance of \( \tilde{v} \) smaller. Then, first the asset liquidation value \( \tilde{v} = \tilde{v} \) is determined exogenously, and next two informed traders acquire the signals respectively. (Trader 1’s signal \( \tilde{v} = \tilde{v} \) and trader 2’s signal \( \tilde{s} = s \) have heterogeneous values each other, unless \( K = 1 \).) Each informed trader believes that the signal he acquired is the real liquidation value of the risky asset.

Similarly to model 1, we assume that the trading strategies of informed traders are linear functions of their signals, denoted \( X_1(\tilde{v}) = \beta_1 \tilde{v} \), \( X_2(\tilde{s}) = \beta_2 \tilde{s} \), respectively. After observing the signal \( \tilde{v} = \tilde{v} \), trader 1 attempts to maximize his expected profit \( (\tilde{v} - \tilde{\rho})\tilde{x}_1 \), conditional on his signal. We assume that one informed trader knows the belief of the other informed trader, each other. So we assume that trader 1 knows trader 2’s signal \( K \tilde{v} \) and considers that trader 2’s trading quantity is denoted as \( \tilde{x}_2 = \beta_2 K \tilde{v} \). On the other hand, after observing \( \tilde{s} = s \), trader 2 attempts to
maximize his expected profit \((\tilde{s} - \tilde{p})\tilde{x}_2\), conditional on his signal. Trader 2 knows trader 1’s signal \(s\) and considers that trader 1’s trading quantity is denoted as \(\tilde{x}_1 = \frac{\beta_1}{K}s\). Liquidity traders’ aggregate order is \(\bar{s}\), and market makers set trading price efficiently conditional on the market order imbalance \(\bar{y} = \bar{x}_1 + \bar{x}_2 + \bar{z}\), as model 1.

Thus, informed trader \(i\) determines the trading quantity taking into account informed trader \(j\)’s trading strategy and market makers’ pricing rule \((i,j=1,2)\). The difference between \((\tilde{v} - \tilde{p})\tilde{x}_1\) and \((\tilde{s} - \tilde{p})\tilde{x}_2\) are caused by the fact that each trader believes his signal to be correct liquidation value, respectively. Actually, trader 1’s belief is rational and if \(K \neq 1\), trader 2’s belief is irrational. Each informed trader attempts to maximize his profit. But if \(K \neq 1\), trader 2’s action of maximizing \((\tilde{s} - \tilde{p})\tilde{x}_2\) may not be rational. If \(K=1\), this model is equivalent to model 1.

### 3.2 Equilibrium

As model 1, we determine \(\bar{x}_1, \bar{x}_2, \bar{p}\), in order to satisfy both the market efficiency condition and the profit maximization condition. This model has an equilibrium as following theorem.

**Theorem 2:** There exists a unique linear equilibrium in which \(X_1, X_2, P\) are linear functions, if and only if \(\frac{1}{5} < K < 2\). The equilibrium trading strategies and the price are of the form:

\[
X_1(\tilde{v}) = \beta_1 \tilde{v} \\
X_2(\tilde{s}) = \beta_2 \tilde{s} \\
P(\bar{y}) = \mu \bar{y}
\]

where the parameters \(\beta_1, \beta_2, \mu\) are given by

\[
\beta_1 = \frac{(2 - K) \phi}{\sqrt{a}} \\
\beta_2 = \frac{(2 K - 1) \phi}{K \sqrt{a}} \\
\mu = \frac{\sqrt{a}}{3 \phi}
\]
and $a=-5K^2+11K-2$.

It should be noted that if $K=1$, this equilibrium is equivalent to the equilibrium in model 1.

**proof:** Similarly to the proof of theorem 1, trader 1’s expected profit conditional on his signal $\tilde{v}$ is given by

$$E[(\tilde{v} - \hat{p})\tilde{x}_1 \mid \tilde{v} = v] = E[(\tilde{v} - \mu(\tilde{x}_1 + \beta_2K\tilde{v} + \tilde{z}))\tilde{x}_1 \mid \tilde{v} = v]$$

$$= (1 - \mu\beta_2K)\tilde{x}_1 - \mu\tilde{x}_1^2$$

The first order condition yields the optimal strategy as follows:

$$\tilde{x}_1^* = \frac{1 - \mu\beta_2K}{2\mu} v$$

Therefore,

$$\beta_1 = \frac{1 - \mu\beta_2K}{2\mu}$$

(7)

Trader 2’s expected profit conditional on his signal $\tilde{s}$ is given by

$$E[(\tilde{s} - \hat{p})x_2 \mid \tilde{s} = s] = E[(\tilde{s} - \mu(\frac{\beta_1}{K}\tilde{s} + x_2 + \tilde{z}))x_2 \mid \tilde{s} = s]$$

$$= (1 - \mu\beta_2K)x_2 - \mu\tilde{x}_2^2$$

From the first order condition,

$$\tilde{x}_2^* = \frac{1 - \mu\beta_1 / K}{2\mu} s$$

Therefore,

$$\beta_2 = \frac{1 - \mu\beta_1 / K}{2\mu}$$

(8)

$\beta_1, \beta_2$ can be solved simultaneously for the given $\mu$, and the solution is given by

$$\beta_1 = \frac{2 - K}{3\mu}$$

(9)
\[ \beta_2 = \frac{2K - 1}{3K\mu} \quad (10) \]

Given \( \beta_1, \beta_2 \), the market efficiency condition implies that
\[
P(\bar{y}) = E[\bar{V} \mid \beta_1\bar{V} + \beta_2\bar{Z}] = \frac{\beta_1 + \beta_2K}{\beta_1^2 + \beta_2^2K^2 + \phi^2} \quad (y)
\]

Therefore,
\[ \mu = \frac{\beta_1 + \beta_2K}{\beta_1^2 + \beta_2^2K^2 + \phi^2} \quad (11) \]

Plugging equation (9) and (10) into (11),
\[ \mu^2 = \frac{-5K^2 + 11K - 2}{9\phi^2} \]

When \(-5K^2 + 11K - 2 > 0\), that is, \(1/5 < K < 2\), there exists a parameter \( \mu \). So,
\[ \mu = \frac{\sqrt{-5K^2 + 11K - 2}}{3\phi} \]

Let \(a = -5K^2 + 11K - 2\), parameters \( \beta_1, \beta_2, \mu \) are given by
\[ \beta_1 = \frac{(2 - K)\phi}{\sqrt{a}} \quad (12) \]
\[ \beta_2 = \frac{(2K - 1)\phi}{K\sqrt{a}} \quad (13) \]
\[ \mu = \frac{\sqrt{a}}{3\phi} \quad (14) \]

For \( \mu > 0 \), the second order condition in informed traders’ optimization problem is satisfied. Q.E.D.

3.3 Properties of Equilibrium

The equilibrium \( X_1, X_2, P \) are determined by the exogenous parameter \( \phi \) and the belief parameter \( K \). And, of course, parameters \( \beta_1, \beta_2, \mu \) also depend on parameter \( K \). Equation (7) gives trader 1’s trading intensity parameter \( \beta_1 \) as a linear Cournot response function to trader 2’s intensity
parameter $\beta_2$. And equation (8) gives trader 2's linear Cournot response function to trader 1's. So, equations (9) and (10) are the solution of Cournot equilibrium. In this model, the belief parameter $K$ must be $1/5 < K < 2$. Otherwise, equilibrium does not exist. If $K \to 1/5$ or $K \to 2$, the market depth is great. If $K = 1$, the equilibrium is equivalent to model 1.

In equilibrium, the trading quantities of informed traders are given by

$$\tilde{x}_1 = \frac{(2 - K) \phi}{\sqrt{a}} \tilde{v},$$

$$\tilde{x}_2 = \frac{(2K - 1) \phi}{K \sqrt{a}} \tilde{s}.$$

Then,

$$E[\tilde{x}_1 | \tilde{v} = v] = \frac{(2 - K)^2 \phi}{3\sqrt{a}} v^2$$

$$E[\tilde{x}_2 | \tilde{v} = v] = \frac{(2K - 1)(2 - K) \phi}{3\sqrt{a}} v^2$$

Therefore, two informed traders' expected profits are as follows:

$$E[\tilde{x}_1] = \frac{(2 - K)^2}{3\sqrt{a}} \sigma_x \sigma_z$$

$$E[\tilde{x}_2] = \frac{(2K - 1)(2 - K)}{3\sqrt{a}} \sigma_x \sigma_z$$

From equations (15) and (16), when $1/5 < K < 1$, trader 1's expected profit is larger than trader 2's, when $1 < K < 2$, trader 2's expected profit is larger than trader 1's, and when $K = 1$, both informed traders' expected profits are given by $\frac{1}{6} \sigma_x \sigma_z$, which is equivalent to the expected profits in model 1. Thus, when $1 < K < 2$, the irrational (underconfident) trader is advantageous. While trader 1's expected profit is always positive, if $1/5 < K < 1/2$, trader 2's expected profit is negative, that is, in this case irrational (overconfident) belief is disadvantageous.
4. The Complete Information Model

4.1 Model 3

In the previous section, we showed that there exists a unique linear equilibrium in the duopoly market of informed traders with the heterogeneous signals. Since we analyzed the model using the method of Kyle and Wang [10], both informed traders’ strategies (intensity parameters) were determined by the liquidity parameter (see equation (9) and (10) in the proof of Theorem 2). Though their signals were heterogeneous, they predicted the same pricing rule. However, since we assume that informed traders believe their private signals to be correct respectively, we can think that signals affect the prediction of the pricing rule set by market makers.

The model 2 is based on Kyle and Wang [10]. We revise the model and investigate the informed traders’ strategies in detail. In model 2 (and Kyle and Wang [10]), when both informed traders determine their trading strategies, they think of the mutual strategies and the pricing rule. Their prior beliefs are different, but their trading intensity parameters are determined by the same liquidity parameter. As their prior beliefs are different and they receive the different signals respectively, we think that their estimation of the market liquidity parameter is different.

The basic setting of the model and the assumptions about signals and prior beliefs are same. The aggregate liquidity trading quantity is also \( \tilde{z} \). And one informed trader has the correct information of the other informed trader’s strategy, each other. When informed trader \( i \) chooses his trading quantity, he takes into account the trader \( j \)’s signal and the pricing rule that trader \( j \) predicts. Then, he maximizes his expected profit based on his signal and the pricing rule that he predicts. That is, we assume the complete information market model. As the model 2, two informed traders’ strategies, denoted \( X_1(\tilde{v}) = \gamma_1 \tilde{v} \), \( X_2(\tilde{s}) = \gamma_2 \tilde{s} \), are linear functions of their signals. And the pricing rule set by market makers, denoted \( P(\tilde{y}) = \nu \tilde{y} \), is also a linear function of the order imbalance:

\[
P(\tilde{y}) = E[\tilde{v} \mid \gamma_1 \tilde{v} + \gamma_2 \tilde{s} + \tilde{z}].
\]

But trader 1 and trader 2 predict the pricing rule:

\[
P_1(\tilde{y}) = E[\tilde{v} \mid \gamma_1 \tilde{v} + \gamma_2 K \tilde{v} + \tilde{z}],
\]

\[
P_2(\tilde{y}) = E[\tilde{s} \mid \frac{\gamma_1}{K} \tilde{s} + \gamma_2 \tilde{s} + \tilde{z}].
\]

In the setting of the model provided above, we analyze the market equilibrium.
4.2 Equilibrium

We determine $\tilde{x}_1, \tilde{x}_2, \tilde{p}$, in order to satisfy both the market efficiency condition and the profit maximization condition. This model has a unique linear equilibrium as following theorem.

**Theorem 3:** There exists a unique linear equilibrium in which $X_1, X_2, P$ are linear functions. The equilibrium trading strategies and the price are of the form:

$$X_1(\tilde{v}) = \gamma_1 \tilde{v}$$
$$X_2(\tilde{s}) = \gamma_2 \tilde{s}$$
$$P(\tilde{y}) = \nu \tilde{y}$$

where the parameters $\gamma_1, \gamma_2, \nu$ are given by

$$\gamma_1 = \frac{\phi}{2}$$
$$\gamma_2 = \frac{\phi}{2K}$$
$$\nu = \frac{2}{3\phi}$$

It should be noted that if $K=1$, this equilibrium is equivalent to the equilibrium in model 1.

**proof:** Trader 1's expected profit conditional on his signal $\tilde{v}$ is given by

$$E[(\tilde{v} - \tilde{p})x_1 | \tilde{v} = v] = E[(\tilde{v} - \nu_1 (x_1 + \gamma_2 K \tilde{v} + \tilde{z}))x_1 | \tilde{v} = v]$$

$$= (1 - \nu_1 \gamma_2 K) \nu_1 x_1 - \nu_1 x_1^2$$

The first order condition yields the optimal strategy as follows:

$$x_1^* = \frac{1 - \nu_1 \gamma_2 K}{2\nu_1}$$

Therefore,

$$\gamma_1 = \frac{1 - \nu_1 \gamma_2 K}{2\nu_1} \quad (17)$$

Trader 2's expected profit conditional on his signal $\tilde{s}$ is given by
From the first order condition, 
\[ x_2^* = \frac{1 - \nu_2 \gamma_1 / K}{2 \nu_2} s \]

Therefore, 
\[ \gamma_2' = \frac{1 - \nu_2 \gamma_1 / K}{2 \nu_2} \] \hspace{1cm} \text{(18)}

\( \nu_i' \), \((i=1, 2)\) is the liquidity parameter that each informed trader predicts. If one informed trader can know the other trader's prediction of the pricing rule, \( \gamma_1', \gamma_2' \) can be solved simultaneously and the solution is given by

\[ \gamma_1 = \frac{2 \nu_2 - \nu_1 K}{3 \nu_1 \nu_2} \] \hspace{1cm} \text{(19)}

\[ \gamma_2 = \frac{2 \nu_1 K - \nu_2}{3 \nu_1 \nu_2 K} \] \hspace{1cm} \text{(20)}

Here, trader 1 considers the pricing rule as follows:

\[ P_1(\tilde{y}) = E[\tilde{y} | \gamma_1 \tilde{v} + \gamma_2 K \tilde{v} + \tilde{z}] \]

\[ = \frac{\gamma_1 + \gamma_2 K}{\gamma_1^2 + \gamma_2^2 K^2 + \phi^2} \tilde{y} \]

Therefore, 
\[ \nu_1 = \frac{\gamma_1 + \gamma_2 K}{\gamma_1^2 + \gamma_2^2 K^2 + \phi^2} \] \hspace{1cm} \text{(21)}

And trader 2 considers the pricing rule as follows:

\[ P_2(\tilde{y}) = E[\tilde{y} | \frac{\gamma_1}{K} \tilde{s} + \gamma_2 \tilde{s} + \tilde{z}] \]

\[ = \frac{\gamma_1 K + \gamma_2 K^2}{\gamma_1^2 + \gamma_2^2 K^2 + \phi^2} \tilde{y} \]

Therefore,

\[ E[(\tilde{s} - \tilde{p})x_2 | \tilde{s} = s] = E[(\tilde{s} - \nu_2 (\gamma_1 \tilde{s} + x_2 + \tilde{z}))x_2 | \tilde{s} = s] \]

\[ = (1 - \frac{\nu_2 \gamma_1}{K})sx_2 - \nu_2 x_2^2 \]
\[ v_2 = \frac{\gamma_1 K + \gamma_2 K^2}{\gamma_1^2 + \gamma_2^2 K^2 + \phi^2} \]  

(22)

So, from equation (21) and (22),

\[ v_2 = K v_1 \]

Then, rewriting equation (19) and (20),

\[ \gamma'_1 = \frac{1}{3 \nu_1} \]  

(23)

\[ \gamma'_2 = \frac{1}{3 \nu_1 K} \]  

(24)

Plugging equation (23) and (24) into (21), then \( \nu_1 \) can be solved,

\[ \nu_1 = \frac{2}{3 \phi} \]  

(25)

So,

\[ \nu_2 = \frac{2 K}{3 \phi} \]  

(26)

Therefore, two informed traders' trading intensity parameters are given by

\[ \gamma'_1 = \frac{\phi}{2} \]  

(27)

\[ \gamma'_2 = \frac{\phi}{2 K} \]  

(28)

Since \( \nu_1, \nu_2 > 0 \), the second order condition in informed traders' optimization problem is satisfied. From the market efficiency condition, the real pricing rule of market makers is given by

\[ P(\tilde{y}) = E[\tilde{v} \mid \gamma_1 \tilde{v} + \gamma_2 \tilde{s} + \tilde{z}] \]

\[ = \frac{\gamma_1 + \gamma_2 K}{\gamma_1^2 + \gamma_2^2 K^2 + \phi^2} \tilde{y} \]

Therefore,

\[ \nu = \frac{\gamma_1 + \gamma_2 K}{\gamma_1^2 + \gamma_2^2 K^2 + \phi^2} \]  

(29)
From equation (27) and (28),
\[ \nu = \frac{2}{3\phi} \]  
(30)
Q.E.D.

In equilibrium, from equation (27) and (28), trader 2’s trading intensity depends on the belief parameter \( K \), on the other hand, trader 1’s trading intensity isn’t affected by the belief parameter. If \( K < 1 \), that is, the belief of trader 2 is overconfident, his intensity becomes large, and if \( K > 1 \), that is, the belief is underconfident, his intensity becomes small. Since we assume the complete information market, each informed trader can know the other informed trader’s strategy. Furthermore, they can consider the mutual prediction of the pricing rule correctly. Practically only the pricing rule that the rational trader predicts is correct. The irrational trader’s prediction of the pricing rule is incorrect, but as he has the information of the rational trader’s action, he can adjust his action. In this model, two traders’ signals perfectly correlate, consequently both informed traders’ trading quantities become same. Therefore, their expected profits are equal.

5. On Parameters and the Expected Profit in Each Model

In model 1, when the noise trading ratio parameter \( \phi \rightarrow 0 \) \((\sigma_z \rightarrow 0)\), the market depth \((1/\lambda)\) and the informed trading intensity \((\alpha_1, \alpha_2)\) become very small. So, for the informed trading, the liquidity trading is essential. If \( K \) is fixed, the same result is shown in model 2 and 3. Then, market makers would provide so much depth that informed traders want to trade more aggressively. In this model the expected profits of two informed traders are given by \( \frac{1}{6}\sigma, \sigma \). So, informed traders’ expected profits are always positive.

In model 2, when we consider \( \phi \) to be constant, if \( K \rightarrow 1/5 \) and \( K \rightarrow 2 \), the liquidity parameter \( \mu \rightarrow 0 \). Though both informed traders’ beliefs are heterogeneous, they recognize the difference of beliefs precisely. Since trader 1’s belief is rational, his expected profit is always positive (from \( 1/5 < K < 2 \), \( E[\tilde{x}_1] = \frac{(2-K)^2}{3\sqrt{a}} \sigma, \sigma_z > 0 \)). On the other hand, except the case of \( K = 1 \), trader 2’s belief is irrational, and if \( 1/5 < K < 1/2 \), his expected
profit \( E[\pi_2] = \frac{(2K - 1)(2 - K)}{3\sqrt{a}} \sigma_v \sigma_z \) is negative.

In model 3, contrast to model 2, when \( 0 < K < 1 \), trader 2's trading intensity becomes large, and when \( K > 1 \), his intensity becomes small. Both traders know the mutual beliefs (signals), and they believe that their own signals are the real liquidation value of the asset, respectively. So, trader 1 determines his trading quantity based on the parameter \( \nu_1 \), taking into account that trader 2's estimation of the liquidity parameter is \( \nu_2 \). Trader 2's action is similar. Then, the rational trader's estimation of the liquidity parameter is correct. However, as this model is the complete information model, the irrational trader can adjust his trading strategy and his expected profit is equal to the rational trader's \( (E[\pi_1] = E[\pi_2] = \frac{1}{6} \sigma_v \sigma_z) \).

Thus, the equilibrium states of model 2 and 3 are quite different. Since, in this paper, we assume the perfect correlation signal, if one informed trader knows the other informed trader's belief, he can know the opponent signal precisely. Consequently, both informed traders can react to the opponent strategies each other, in model 3, two informed traders' trading quantities become equal. But in the real market their mutual estimation of the signals may includes the errors. So, they can't adjust their trading strategies perfectly.

6. Conclusion

In this paper, we have considered the effect of signals in informed trading. Following the method of Kyle [8] and Kyle and Wang [10], we were able to analyze the informed trading duopoly market. In model 1, we derived the existence of a unique linear equilibrium in the market where two informed traders with the same private signal. And we showed that in equilibrium the trading intensity parameters of two informed traders are the solution of Cournot equilibrium. Both informed traders' intensities are equal, so their expected profits are also equal and always positive. Then, some liquidity traders suffer losses.

In model 2 and 3, we dealt with the case that two informed traders' private signals are heterogeneous respectively. We considered the difference of private signals caused by the each trader's prior belief, and we introduced the belief parameter \( K \). Unless \( K = 1 \), informed trader 2's belief is irrational. In
model 2, two informed traders know the opponent belief precisely each other. Then, if $K$ satisfies the condition, there exists a unique linear equilibrium, and the belief parameter $K$ influences both informed traders' trading intensities. The rational trader's intensity is always positive and his expected profit is also positive, because he can react to the irrational trader's trading strategy. In model 3, we provided the revision of model 2, and showed that the difference of signals (beliefs) affected not only the trading intensity but also the prediction of the pricing rule. That is, two informed traders have the heterogeneous estimation of the market liquidity parameter. In this model, however, both traders recognize the mutual trading actions, they can choose the optimal trading strategies. Therefore, both informed traders' expected profits are positive and equal. And we also showed that in model 2, when the irrational trader's belief was underconfident, his intensity became large, on the other hand, in model 3, when his belief was overconfident, his intensity became large. So, model 3 is realistic about the informed trader's action.

In this paper it is important for the informed trader to understand not only his own belief but also the belief of the opponent. If one informed trader knows the other informed trader's belief, he can predict the signal that his opponent acquired well. When one informed trader determines his own order, he must think of the other informed trader's strategy. Therefore, the accuracy of the prediction influences the trading result. Moreover, if they consider the opponent prediction of the pricing rule, then they can adjust their own trading strategies perfectly.

We have assumed that two informed traders' heterogeneous signals are expressed by scalar multiple. So, from now on, we will able to consider another form of signals and trading strategies. For example, it is possible to assume that both informed traders can't know the real opponent signal's form but they can form the belief of the opponent signal's distribution. And when they construct their trading strategies, we may be able to make the linear trading strategies including the both traders' signals. For another extension of this model, it may be possible to make the model to be Bayesian game.

References


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