Abstract

This paper presents a methodology that solves the global newsvendors problem. It determines the optimal transfer price and the optimal order level of a product at the secondary market simultaneously. The optimal transfer pricing in the global supply chain management is a critical issue, along with quotas, trade barriers, duty drawbacks, etc., related to international trade. Different transfer prices result in significantly different global total after-tax profits for multinational corporations. The optimal order level, traditionally, has also been an important topic in the global newsvendors problem. The methodology proposed in this study finds both the optimal order level and the optimal transfer price iteratively.

Keywords: Global supply chain management; Newsboy problem; Transfer pricing

1. Introduction

Advanced information technology, decreasing tariffs, improvements in transportation, Free Trade Agreements, the European Union (EU), greater access to wider variety of products and services have all lead to strong economic globalization. Consequently, manufacturing companies must provide a variety of products and services to meet the needs of customers in order to survive (Mentzer, 2001). Therefore, the strategies of corporations have changed from the mass production with a limited number of standardized products and services, to mass customization to provide products and services meeting individual customer requirements, while maintaining high quality, low unit cost, and short lead time (Cohen and Moon, 1989; Reddy and Reddy, 2001). To reduce unit production cost, many corporations have constructed manufacturing factories in foreign countries to save on labor, raw materials, or transportation. Due to geographical issues, the lead-times and order cycle times of products and services have been getting longer. Therefore, most global corporations have focused on re-engineering and optimizing all internal and external processes from the acquisitions of raw materials to the deliveries of final products in an effort to increase productivity.

One of the approaches of re-engineering is the SCM (Supply Chain Management) systems in the late 1990s that allows companies to innovate their processes by integrating and optimizing all their processes, from suppliers through internal business processes to customers. The SCM can be defined as an integrative approach to manage the flows of distribution channels from the suppliers to the ultimate users (Cooper and Elram, 1993). The SCM reduces costs and improves services based on collaborations with customers and suppliers throughout the supply chain.

In the SCM, the level of product availability is a critical factor. The high level of product availability can guarantee high responsiveness to customers and it builds a strong company image. It may, however, keep unnecessary inventories which lead to high inventory-holding costs or cash-flow binds. On the other hand, the low level of product availability can give damage future business, and thus tarnish the company's image. The level of product availability is also a critical factor in the GSCM (Global Supply Chain Management), which involves the supply chains of globally dispersed suppliers and markets (Cohen and Lee, 1989; Arntzen, 1995; Thomas and Griffin 1995). The Global Newsvendors Problem is one of the GSCM problems that determine the optimal level of seasonal product availability, transferred from one company to a subsidiary company under the same firm located in a foreign country (Kouvelis and Gutierrez, 1997).

Transfer pricing is one of the most important issues in GSCM because it directly determines the level of profits. Transfer price is defined as the price that a selling department, division or subsidiary of a company charges for products and services supplied to a buying department, division or subsidiary of the same firm (Abdallah, 1989). How to establish an optimal transfer price has been a controversial issue over the last 40 years in accounting and economics (O’Connor, 1997; Choi et al. 1999).

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The objective of this study is to solve the Global Newsvendors Problem that determines the optimal transfer price and the optimal level of product availability by identifying the relationship between the level and the price.

2. Literature Review

2.1 The Classical Newsboy Problem

The problem of determining the optimal level of the product availability is one of the stochastic inventory problems. Nahmias (1997) called this problem a ‘Newsboy model’. This model is based on the assumption that a single product is to be ordered at the beginning of a period and the product has a useful life of exactly one single planning period. Also it is assumed that the demand during the period is a continuous non-negative random variable with a density function f(x) and a cumulative distribution function F(x). The optimal level of product availability Q* is obtained by the following equation.

\[ F(Q^*) = \frac{C_o}{C_o + C_u} \]  

where  
\[ C_o = \text{cost per unit of remaining inventory at the end of the period (overage cost)} \]  
\[ C_u = \text{cost per unit of unsatisfied demand, negative ending inventory (underage cost)} \]

2.2 Global Newsvendors Problem

In today’s market, corporate managers face new challenges in determining the optimal level of product availability of seasonal products or fashionable items such as style goods. Previously, it was assumed that a single style good has a useful life of exactly one planning period. However, today’s global markets allow the single item to have another useful life in other foreign markets. As Kouvelis and Gutierrez (1997) pointed out, from a production standpoint, the opportunity to exploit the difference in timing of the selling season of geographically dispersed markets for “style goods” is important for improving the firm’s profitability. They proposed a mathematical model to determine the production quantity, y*, to improve the profitability as follows:

\[ F(y^*) = \frac{C_u - t - T}{C_u - s} \]  

where  
\[ C_u = \text{underage cost at the secondary market} \]  
\[ s = \text{salvage value at the secondary market,} \]  
\[ t = \text{transfer price (given),} \]  
\[ T = \text{transportation cost per unit from the primary market to the secondary market} \]

This model assumes that transfer price is given and fixed. However, in real life situations, transfer price as well as production quantity should be determined simultaneously in order to maximize the profitability.

2.3 Transfer Pricing

Since the late 1950s, engineers, accountants and economists have given attention to establishing an appropriate transfer price (TP), because different TP yields significantly different total after-tax profits of multi-national corporations (MNCs). The total after-tax profits of MNCs can be maximized by shifting the taxable incomes to other foreign countries that offer tax incentives.

2.3.1 The Vidal and Goetschalckx Model

Vidal and Goetschalckx (2001) introduced a linear mathematical model to decide the optimal transfer price and present a simple numeric example with a single-product with two subsidiaries. Their model is a non-linear model; however, they solve the problem with linear programming after substituting a non-linear term into an artificial variable. The problem is as follows.

Product q is assembled at the country A, and the amount x of product q is transferred from the country A to the country B for sales at the market price S. If the transfer price of the product q is t, the firm at country A has the revenue tx. In the country B the revenue is Sx if all the amount x of the product q are sold in the market. If variable cost v is required to produce one unit of the product q, the total variable production cost of the country A is vx, while at the country B the procurement cost tx should be paid. If the fixed costs of the country A and B are Fa, Fb, respectively, the production cost of the country A is vx+Fa. If the transportation cost of one unit of the product q is T and the responsible portion of the country A is (0 ≤ ≤ 1), the transportation cost of the country A is Tx and the transportation cost of the country B is (1- )Tx. In the country B, if the rate of import duty is , then the import duty tTx must be paid. Therefore, the net income before-tax in the country A is x- Tx-(vx+Fa), and the net income before tax in the country B is Sx-tx-Fb-(1- )Tx - tTx. Also, income taxes must be paid in both countries. Let the income tax rates of the country A and the country B be TAXa and TAXb, respectively. The net income after-taxes of the country A and B are given in Table 1.

A net income before-tax can be either negative or positive. In case of the positive net income before-tax, an income tax must be paid; however, in case of negative net income before-tax, there is no income tax. Let’s consider an example problem introduced by Vidal and Goetschalckx (2001).

2.3.2 Example Problem by Vidal and Goetschalckx

Let’s suppose IBTA and IBTB are net income before taxes at the country A and the country B, respectively.
Table 1. After-Tax Profit Calculations of Global Corporations

<table>
<thead>
<tr>
<th>Detail</th>
<th>Country A</th>
<th>Country B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>tx</td>
<td>tx</td>
</tr>
<tr>
<td>Procurement costs</td>
<td>αTx</td>
<td>(1-α)Tx</td>
</tr>
<tr>
<td>Transportation costs</td>
<td>vx</td>
<td></td>
</tr>
<tr>
<td>Other variable costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Import duties</td>
<td>δTx</td>
<td></td>
</tr>
<tr>
<td>Fixed costs</td>
<td>Fa</td>
<td>Fb</td>
</tr>
<tr>
<td>Net Income Before Tax</td>
<td>TAx(tx-vx)Fx</td>
<td>TAx(S-(1+)tx-Fb)</td>
</tr>
<tr>
<td>Taxes</td>
<td></td>
<td></td>
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<tr>
<td>Net Income After Tax</td>
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</tbody>
</table>

For simplification, transportation cost is not considered (i.e., T = 0) in this example problem. The data for the problem is as follows:
- δ: Import duty charge = 12%
- Fa: Fixed cost at country A = $20,000
- Fb: Fixed cost at country B = $120,000
- S: Market price of the product in country B = $20/unit
- TAXa: Income tax rate in country A = 34%
- TAXb: Income tax rate in country B = 50%
- v: Variable cost in country A = $7/unit
- Demand = 20,000 units

Let us introduce non-negative variables A+, A-, B+ and B- to represent IBTA and IBTB with nonnegative values (i.e., IBTA = A+ - A- and IBTB = B+ - B-). The initial mathematical model is given in (P1). The objective function of this model is to maximize global corporate profits.

(P1) Max Profit = 0.66 A+ - A- + 0.5 B+ - B-

Subject to
- 7x - A+ - A- = 20,000
- 1.12 tx + 20x + B+ - B- = 120,000
- 11 ≤ t ≤ 13
- x ≤ 20,000

Problem (P1) is a non-linear model that includes a non-linear term tx in the first two constraints. Vidal and Goetschalckx (2001) solved this problem using linear programming after substituting the term tx with an artificial variable y in (P2).

(P2) Max Profit = 0.66 A+ - A- + 0.5 B+ - B-

Subject to
- y - 7x - A+ - A- = 20,000

This model still has one critical problem. The third constraint is a boundary condition of transfer price and the last constraint is an upper boundary for the demand. Without the boundary conditions, the model is unbounded. The profit function is non-decreasing, that means the higher the quantity is allocated, the higher the profit is guaranteed as shown in Figure 1. The proof is as follows.

When the amount x of the product q is transferred from the country A to the country B, total after-tax profit (x) is

\[(x) = [1-Taxa](tx-vx-Fa)+[1-Taxb](Sx-(1+)x-Fb)\]  (3)

And when the amount x+ε of the product q is transferred from the country A to the country B, total after-tax profit π (x+ε) is

\[[1-Taxa](t(x+\varepsilon)-v(x+\varepsilon)-Fa)+[1-Taxb](S(x+\varepsilon)-(1+)x+Fb).\]

Hence, if (x+) > (x), then it can be said that the total after-tax profit function is a non-decreasing function when > 0. The difference between (x+) and (x) is

\[(x+) - (x) = [1-Taxa]t-v+1-Taxb][S - (1+)t-Fb] > 0\]

The term t-v is the margin of the country A, and the term S - (1+) is the margin of the country B. Since the margins in both countries are non-negative, it can be concluded that (x+) > (x).

3. Proposed Algorithm for Optimal Transfer Pricing

Instead of using linear programming, if the demand is known and fixed, it is possible to obtain the optimal transfer price explicitly by identifying the relationship of the quantity to be transferred and the transfer price. Since the upper bound of the demand is 20,000 units in the example problem, the global after-tax profit is

\[[1 - Taxa](tx - vx - Fa) + [1 - Taxb](Sx - (1+)x - Fb)\]

= 0.66(20,000t - 160,000)+ 0.5(-22,400t + 200,000).

The Figure 2 shows that the optimal transfer price is $12.50 when the transferred volume given is 20,000 units. The function of total after-tax profits is the line with (-) which looks like a trapezoid. The lines with and indicate the after-tax profit function of the country A and of the country B, respectively. The after-tax profit line of the country A has a seam on the horizon axis because the income tax rate of the country A, TAX a is applied.
Similarly the after-tax profit line of the country B also has a seam on the same horizon axis due to income tax rate of the country B, TAXb. The after-tax profits of the country A and the country B can be calculated as follows.

The after-tax profits of the country A =

\[ [1 - \text{Taxa}] \text{tx} - vx - Fa \quad (4) \]

The after-tax profits of the country B =

\[ [1 - \text{Taxb}] Sx - (1+tx) - Fb \quad (5) \]

The solid lines in Figure 3 show the total profit of both countries A and B. From the Figure 3, the important fact is that at one of the two seam points the maximum of the after-tax profit is achieved. It is noted that at the seam points, either one of the after-tax profits of country A or B becomes zero. In other words, the maximum global profits can be achieved at the points where after-tax profit of country A or B becomes zero. Therefore the optimal transfer price can be determined by solving the two cases of zero after-tax profits of country A and B. The transfer price where the profit of the country A is equal to zero can be calculated as follows.
3. The General Linear Functions of Total After-Tax Profit

The after-tax profit of the country $A$

$$= [1 - \text{TAX}_a] (t - v)x - F_a = 0$$

$$t = \frac{F_a}{x} + v \quad (6)$$

Similarly, the transfer price where the profit of country $B$ is equal to zero is

The after-tax profit of the country $B$

$$= -[1 - \text{TAX}_b] (1 + \delta)xt + (Sx - F_b) = 0$$

$$t = \frac{Sx - F_b}{(1 + \delta)x} \quad (7)$$

Accordingly, the after-tax profit of the country $A$ is

$$(1 - \text{TAX}_a) [(\frac{Sx - F_a}{(1 + \delta)x} - v)x - F_a]$$

The after-tax profit of the country $B$ is

$$(1 - \text{TAX}_b) [(S - (1 + \delta)(\frac{F_a}{x} + v))x - F_b]$$

Thus, the optimal transfer price is the price with

$$\text{Max} \, ((1 - \text{TAX}_a) [(\frac{Sx - F_a}{(1 + \delta)x} - v)x - F_a]),$$

$$\text{Max} \, ((1 - \text{TAX}_b) [(S - (1 + \delta)(\frac{F_a}{x} + v))x - F_b])$$

4. Optimal Level of Product to Transfer and Optimal Transfer Price

In real life situations, the actual consumption is not known in advance. Thus, the quantity transferred to the country $B$ will turn out to be either under- or over-supplied.

There are costs related to under- or over-supply. The underage cost is the difference of selling and transfer prices. The overage cost is the difference of transfer price and salvage value. Using the underage and overage cost information, it is possible to obtain the optimal level of product transferred to the country $B$. Once the level to transfer is determined, a new optimal transfer price can be recalculated. The new transfer price changes the underage and the overage cost information. It also changes the optimal level to transfer and so on. If the transfer prices and the levels to transfer obtained at iterations converge, the values are better estimates of the optimal level transfer and the optimal transfer price.

Let us assume that the country $A$ has the maximum of the after-tax profit. The optimal level of product to transfer $x^*$ in the newsboy problem in a global market can be obtained as follows.

$$x^* = F^{-1} \left[ \frac{C_a}{C_v + C_o} \right] = F^{-1} \left[ \frac{S - t}{S - v} \right] \quad (8)$$

By plugging $x^*$ into the equation (4), the maximum after-tax profit will be calculated as below.

$$(1 - \text{TAX}_a) [(t - v)(F^{-1}(\frac{C_a}{C_v + C_o}) - F_a)]$$

$$= (1 - \text{TAX}_b) [(t - v)(F^{-1}(\frac{S - t}{S - v}) - F_b)] \quad (9)$$

Let $t^*$ be the value of $t$ that maximizes the equation (9). Then by plugging $t^*$ into equation (8), the new value of $x^*$ will be obtained. By repeating these steps, if the values of $t$ and $x$ are converged, the optimum values of the after-tax profits maximization will be achieved.

Theorem:
Let $t_0$ be an initial guess of the transfer price and $t_1$ be a new transfer price obtained after the first iteration. If $(S - t_1) > 0.5$ and $(S - t_0) > 0.5$, then $t$ and $x$ are converged.

Proof is given in Appendix A.

5. Illustration

The data for this illustration is the same as the ones provided by Vidal and Goetschalckx, excepting the demand information. Let’s suppose that the demand at the secondary market is normally distributed with mean of 20,000 units and standard deviation of 1,000 units. For an initial level of product to transfer $x_0$, the mean is used as a starting seed value of $x$.

According to the proposed algorithm, the initial optimal transfer price for the optimal level of product to transfer to the secondary market can be determined by setting the net before-tax profits of the primary market and the secondary market to zeros as follows:

At the primary market,

\[
t = \frac{Sx - F_o}{(1 + \delta) x} = \frac{20(20,000) - 120,000}{(1 + 0.12)20,000} = 12.5
\]

At the secondary market,

\[
t = \frac{F_o + \sigma}{x} = \frac{20,000 + 7}{20,000} = 8
\]

Accordingly, the global total after-tax profit of the country $A$ is

\[
(1 - \tau_A) \left[ \left( \frac{Sx - F_o}{(1 + \delta)x} \right) x - F_o \right]
\]

\[
= (1 - 0.34) \left[ (12.5 - 7) \cdot 20000 - 20000 \right] = 59,400.
\]

The global total after-tax profit of the country $B$ is

\[
(1 - \tau_B) \left[ \left( S - (1 + \delta) \left( \frac{F_o + \sigma}{x} \right) \right) x - F_o \right] = (1 - 0.5) \left[ \frac{(20000 - 20000) - 120,000}{20,000 - 120,000} \right] = 50,400.
\]

Thus, the optimal transfer price is $12.5$ with the maximum of $(59,400, 50,400)$.

Note that the optimal order quantity, 20,000 units should be adjusted because the production cost should be replaced with the transfer price and the transportation cost when the optimal order quantity is calculated. Thus, the updated optimal order quantity can be calculated as follows. Transportation cost is assumed as zero for convenience.

\[
F_o(x^*_1) = \frac{S - t}{(S - \nu)20 - 12.5} = 0.75 \quad \text{and} \quad x^*_1 = \mu + \sigma \cdot Z_{0.75} = 20,000 + 1,000(0.67449) = 20,674.5
\]

Again, this new optimal order quantity will change the initial transfer price, $12.50$ into the newly updated transfer price by the proposed algorithm. If the new optimal transfer price is almost same as the previous one, which indicates that optimal order quantity is converged. The final optimal order quantity is 20,624 units and the optimal transfer price is $12.66$. The global total after-tax profit is increased to $63,874$. Table 2 shows the convergence of the optimal level and the transfer price.

6. Conclusions

The determination of the level of product to transfer and the transfer price is an important decision for managers of MNCs. Many researchers have extensively studied optimizing the level of product availability (or to transfer) for the secondary market and also tried to establish an optimal transfer price. However, there has been no meaningful effort to optimally set up the two critical factors simultaneously, until Vidal and Goetschalckx proposed a mathematical programming approach.

In this study, a methodology that finds an optimal transfer price explicitly has been successfully developed. It reduces a considerable amount of calculations for multinational corporations in obtaining optimal transfer price. The methodology clearly identifies the relationship of the total after-tax profit of the multinational corporation and the level of product to transfer from the primary market to the secondary market, while Vidal and Goetschalckx solve the nonlinear problem via linear programming by substituting a nonlinear term (i.e., product of two variables) with an artificial variable. The mathematical model suggested by Vidal and Goetschalckx is unbounded. That means all the items transferred to the secondary market are assumed to be sold out. The model does not consider the costs related to shortage and overage of the product availability at the secondary market. The methodology proposed in this study finds an optimal level of product to transfer and transfer price by considering overage and underage costs after an optimal transfer price is obtained by means of iterative process.

For further research, investigating the more realistic problem with multi-item transfer prices and levels to transfer for multinational corporations is recommended. It is a challenging problem due to the complexity of the after-tax profit formula. It is also noted that a new after-tax profit formula is given in Appendix A.
profit estimator that considers costs related to overage and underage of the product availability needs to be developed. If successful, it may take care of the unrealistic unboundedness of the problem.

References


Appendix A.

Theorem:

Let \( t_0 \) be an initial guess of the transfer price and \( t_1 \) be a new transfer price obtained after the first iteration. If \( \frac{S - t_1}{S - v} > 0.5 \) and \( \frac{S - t_0}{S - v} > 0.5 \), then \( t \) and \( x \) are converged.

Proof:

If \( x_{n-2} < x_o \) and \( x_{n+1} < x_{n-1} \), where \( x \) is normally distributed, then \( x \) converges to a point when \( n \) increases to infinity.

In order to get \( x_{n-2}, x_{n-1}, x_n \), and \( x_{n+1}, t_{n-3}, t_{n-2}, t_{n-1}, t_n \) and \( t_n \) must be calculated in advance.

In order to get \( t_{n-3}, t_{n-2}, t_{n-1}, t_n, x_{n-3}, x_{n-2}, x_{n-1}, x_n \), and \( x_n \) must be calculated in advance.

Ultimately, the relationship between \( x_{n-2} \) and \( x_{n+1} \) is determined by the relationship of \( x_2 \) and \( x_0 \), and the relationship between \( x_{n-1} \) and \( x_{n+1} \) is determined by the relationship of \( x_3 \) and \( x_1 \). Thus, if \( x_2 > x_0 \) and \( x_3 < x_1 \), then it can be said that \( x \) converges to a point when \( n \) increases to infinity.

Let \( x_0 = \mu \), as an initial guess of optimal level of product transferred. Then \( t_0 \) can be obtained. Using \( t_0 \), the value \( x \) (i.e., \( x_i \)) will be updated.

Repeating these steps.

If \( x_2 > x_0 \) and \( x_3 < x_1 \) where \( x \) is normally distributed, then it can be said that \( t \) and \( x \) are converged to \( t^* \) and \( x^* \), respectively.

\[
x_{3} - x_{1} = \left[ \mu + \sigma \cdot Z \left( \frac{S - t_{2}}{S - v} \right) - \mu - \sigma \cdot Z \left( \frac{S - t_{0}}{S - v} \right) \right] = \left[ \sigma \left( Z \left( \frac{S - t_{2}}{S - v} \right) - Z \left( \frac{S - t_{0}}{S - v} \right) \right) \right]
\]

In order to be \( x_3 - x_1 < 0 \),

\[
\left( Z \left( \frac{S - t_{2}}{S - v} \right) - Z \left( \frac{S - t_{0}}{S - v} \right) \right) < 0 \quad \text{must be less than zero}.
\]

In order to be \( \left( Z \left( \frac{S - t_{2}}{S - v} \right) - Z \left( \frac{S - t_{0}}{S - v} \right) \right) < 0 \),

\[
\left( \frac{S - t_{2} - S - t_{1}}{S - v} \right) \quad \text{must be less than zero}.
\]

In order to be \( \left( \frac{S - t_{2} - S - t_{1}}{S - v} \right) < 0 \), \( t_2 \) must be greater than \( t_1 \).

\[
t_2 - t_1 = \frac{Sx_1 - F_b}{(1 + \delta)x_1} - \frac{Sx_0 - F_b}{(1 + \delta)x_0} = \frac{F_b}{(1 + \delta)} \left( x_0 - x_1 \right)
\]

In order to be \( t_2 - t_1 > 0 \), \( x_0 \) must be less than \( x_1 \).

\[
x_1 - x_0 = \left[ \mu + \sigma \cdot Z \left( \frac{S - t_{0}}{S - v} \right) - \mu \right] = \left[ \sigma \cdot Z \left( \frac{S - t_{0}}{S - v} \right) \right]
\]
In order to be \( x_2 - x_0 > 0 \),
\[
\frac{S - t_0}{S - v} > 0.5
\]

\[
x_2 - x_0 = \left[ \mu + \sigma \cdot Z \left( \frac{S - t_0}{S - v} \right) - \mu \right]
\]

In order to be \( x_2 - x_0 > 0 \),
\[
\frac{S - t_0}{S - v} > 0.5
\]