Coordinated Ordering and Pricing Decisions for a Short-life-cycle Product in a Three-stage Supply Chain with Capacity Constraint

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Abstract

This paper studies supply chain coordination in a decentralized supply chain which consists of a distributor, a manufacturer, and an OEM. The distributor meets price-sensitive random demand of a product with a short life cycle. The manufacturer, having limited capacity, receives the order from the distributor; when the distributor's order exceeds the manufacturer's capacity, the manufacturer outsources insufficient portion of production to the OEM. We develop a mechanism to coordinate the supply chain members' decisions in order to improve the total supply chain profit. We consider two coordination scenarios: one is that the OEM does not participate in the coordination, the other is that the OEM participates in the coordination. Each member's optimal decisions in the absence and presence of coordination will be derived. Then, we provide a numerical study to analyze the performance of coordination, and explore the managerial insights for the members in the supply chain.

Keywords: Coordination; Ordering and pricing policies; Supply chain management

1. Introduction

For effective supply chain management, matching supply with demand is a critical issue (Ellarm and Cooper, 1990; Kim et al, 2002). However, most of the supply chain models involve uncertainties which may be on the supply or demand side. In particular, in the presence of stochastic market demand, it is difficult to build matching production capacity to cover every possible demand realization. Outsourcing is a good way to solve poor match between demand realized and available production capacity, because it increases a manufacturer's flexibility of capacity. Yet, there are numerous challenges for a supply chain to successfully implement an outsourcing partnership, and these challenges are centered on the effective coordination of decentralized supply chains (Zhao and Wang, 2002). Many studies (Boyac and Callego, 2002; Chen and Xu, 2001, Kim 2000; Kouvelis and Milner, 2002; Lee, 2001; Su, 2004; Weng, 1999) indicate that coordination mechanisms can be used to promote the performance of decentralized supply chains, and increase supply chain members' profits.

According to (Cachon and Lariviere, 2005), a videocassette retailer, Blockbuster Inc., has successfully adopted the revenue-sharing mechanism to improve market share and alleviate the customers' discontents of the poor availability of new release videos. In 1998, Blockbuster decided to enter into revenue-sharing agreements with the major studios in order to purchase more new release video tapes through paying roughly 30% to 45% of its rental income in exchange for a reduction in the initial price per tape (from $65 to $8). In a year after starting revenue sharing, Blockbuster increased its overall market share from 25% to 31% and its cash flow by 61%.

This paper studies a revenue-sharing coordination mechanism in a three-stage supply chain consisting of an Original Equipment Manufacturer (OEM), a manufacturer, and a distributor. The distributor meets price sensitive random demand, and places the order to the manufacturer. The manufacturer meets price sensitive random demand, and places the order to the OEM. The manufacturer has a finite production capacity, and outsources insufficient capacity to the OEM. All the supply chain members make pricing and ordering policies before the selling season: the distributor determines her unit sales price and ordering quantity placed on the manufacturer; and the manufacturer and the OEM determine their unit sales prices charged to the distributor and to the manufacturer, respectively. Our coordination mechanism coordinates the members' ordering and pricing policies in order to improve the total supply chain profit. The coordination can be divided into two categories: one is the OEM participates in the coordination, the other is that the OEM does not. As a result, we not only show how coordination improves the total supply chain profit, but also investigate the performances of different categories of coordi-
nation. The overall ordering and price procedure will be presented in Section 2.

Many studies have focused on the development of coordination mechanisms to improve total supply chain profits or performances (Cachon and Lariviere, 2005; Giannoccaro and Pontrandolfo, 2004; Tjader et al., 2004; Weng, 1999; Yao and Chiou, 2004; Zimmer, 2002). Weng (1999) pointed out a coordination mechanism that the manufacturer's unit sales price is equal to the manufacturer's unit cost, and a fixed payment from the distributor to the manufacturer determines the allocation of increased profit in the presence of coordination. Weng also indicated that the profit of the supply chain will be improved by using the coordination mechanism to stimulate the distributor to increase his order quantity. However, Weng assumed the manufacturer has infinite capacity, and such an assumption is rather unrealistic in practice. Weng and McClurg (2003) developed supplier-buyer coordinated ordering policies in a two-stage system, in which the buyer meets random demand and places orders to the supplier, whose delivery time is uncertain; they found that coordination will lead to increased system profits. Weng (2004) developed a generalized newsvendor model to analyze an incentive policy that the manufacturer may use to induce the buyer to order the coordinated quantity, when the buyer operates to meet random demand of one product with a short life cycle. Yao and Chiou (2004) considered an integrated supply chain model which includes one vendor and multiple buyers, and developed a replenishment coordination model to minimize the vendor's total annual cost subject to the maximum cost that the buyer may be prepared to incur. Giannoccaro and Pontrandolfo (2004) developed a revenue-sharing mechanism to coordinate a three-stage supply chain, and discussed that their model allows the system efficiency to be improved as well as it could. Cachon and Lariviere (2005) demonstrated that revenue sharing is a very attractive supply chain coordination contract. They studied revenue-sharing contracts in a general supply chain model with revenues determined by each retailer's purchase quantity and price; then, they compared revenue-sharing to a number of other supply chain contracts, and explained why it is not prevalent in all industries. The above studies focus on deciding members' ordering and pricing policies (such an issue also discussed in Konn and Ishii, 2005), and assume that manufacturers' or suppliers' capacity constraints are infinite. This study, in contrast, takes into account a manufacturer's capacity constraint, and uses coordination mechanisms to stimulate more market demand. As such, this study differs from a closely-related study (Weng 1999) in that this study relaxes the assumption of the manufacturer with infinite capacity and considers the coordination with the OEM.

Outsourcing strategy can be found in reducing firms' significant investments in productive facilities, warehouse facilities, and transportation equipment. A supply chain with rapid technological change and uncertain demand usually uses outsourcing strategies to improve flexibility and competition. De Kok (2000) showed that how a production facility chooses its capacity strategy when the demand exceeds the production capacity. He proposed a model that focuses on the choice between two types of capacity policies: a fixed capacity reservation policy and a policy which outsources excess capacity demand. Both policies are useful in different situations. Kim (2003) investigated a model that the manufacturer outsource his assembly operations to two different contract manufacturers. In Kim's model, two contract manufacturers have different improvement capabilities and different prices. The manufacturer chooses how many units should be outsourced to each contract manufacturer, and what the processing level the semi-finished units should be, when returned from the contract manufacturers, in order to reduce the supply cost. Güllü et al. (1999) pointed out that in most of the production models, uncertainties may occur on the demand side or supply side. In light of outsourcing, manufacturers usually make contracts with their suppliers (contract manufacturers) in order to gain more benefits. Zimmer (2002) proposed that the manufacturer makes contract with the OEM to avoid shortage situations, and showed that the contract indeed increases the profits of both the manufacturer and the OEM. Therefore, we adopt outsourcing strategy into the supply chain with uncertain demand in order to deal with the manufacturer's insufficient capacity.

The remainder of this paper is organized as follows. Section 2 presents the uncoordinated model, formulates each member's profit function, and derives each member's optimal ordering and pricing policies. In Section 3, the coordination mechanism will be proposed, and each member's optimal policies in the presence of coordination will be derived. Section 4 provides a numerical example to compare the total supply chain profit in the absence and presence of coordination. Parameter analysis will be used to show how the important parameters vary the performance of coordination. Finally, Section 5 concludes with a brief summary.

2. The Model

The market demand faced by the distributor is stochastic and price-sensitive. We assume the probability density function (pdf) of the price-sensitive random demand is \( f(x | p_j) \, \text{and the expected market demand of} \, f(x | p_j) \, \text{is} \, m(p_j) = D \, p_j^{k-1}, \) where \( D > 0 \) is the scaling constant and \( k > 1 \) is the constant price elasticity. Such an expected market demand function has been adopted
in many literatures for studying price sensitive demand (Ray et al., 2005; Weng, 1999).

In the absence of coordination, there is no information sharing within the supply chain, and each member’s objectives are to maximize his own expected profit. The business processes of this supply chain model are as follows. First, the manufacturer decides his unit sales price \( P_m \) by maximizing his predicted profit, in which a prior function to forecast the distributor’s order quantity is developed. Then, the manufacturer provides \( P_m \) for the distributor. Second, by considering \( P_m \) and the price-sensitive random demand, the distributor determines his order quantity \( Q_d \) and his unit sales price \( P_d \) so as to maximize his own expected profit. Then, the distributor orders \( Q_d \) units from the manufacturer. Finally, after receiving the distributor’s order quantity \( Q_d \), the manufacturer decides his outsourcing order quantity \( Q_m \) by considering his finite capacity \( u \) and \( Q_d \). If the manufacturer has sufficient capacity \( Q_d \leq u \), he will produce all the distributor’s order quantity; otherwise, the manufacturer will outsource insufficient production capacity \( Q_m = Q_d - u \) to the OEM with the unit sale price \( P_o \).

We now discuss each member’s cost structure and profit function which are used to determine his ordering or pricing policy. In this paper, the distributor has control over the ordering quantity \( Q_d \) and the sales price \( P_d \), and the distributor’s ordering policy will be influenced by his attitude toward risk. Let \( z \) denote the distributor’s attitude toward risk, and the relationship between \( z \) and the distributor’s ordering policy is that \( Q_d = z \times m(P_d) \); for example, a risk-averse distributor will have a small value of \( z \), and his order quantity will also be small. We assume that there is no salvage value or disposal cost of the unsold unit at the end of the selling period; similarly, there is no shortage cost of the unsatisfied demand. Wang et al. indicated that zero salvage or zero disposal cost assumptions appropriately reflect the reality for short-life-cycle products. Hence, the distributor’s profit is equal to his total revenue minus his cost:

\[
\Pi_d(P_d, Q_d) = P_d \min \{ x, Q_d \} - P_m Q_d. \tag{1}
\]

The manufacturer’s objective is to offer his unit sales price \( P_m \) to the distributor and to produce the distributor’s order quantity. We assume that the OEM’s unit sales price is larger than the manufacturer’s unit production cost \( c_o \) (i.e., \( P_o > c_o \)) so that the manufacturer will firstly decide to self-produce the distributor’s order quantity. We also assume that the manufacturer’s unit sales price is larger than the OEM’s unit sales price \( P_o \) (i.e., \( P_m > P_o \)) so that the manufacturer is willing to outsource his insufficient capacity to the OEM. Hence, the manufacturer is equal to his total revenue minus his cost and outsourcing cost:

\[
\Pi_m(P_m) = P_m Q_d - c_m \min \{ Q_d, u \} - P_o [Q_d - u]^+. \tag{2}
\]

The OEM produces each unit at cost \( c_o \), and sells for the manufacturer at unit sales price \( P_o \). The parameters \( c_o \) and \( P_o \) are deterministic. The OEM’s profit is equal to that the total selling revenue minus the total production cost:

\[
\Pi_o = (P_o - c_o)[Q_o]^+. \tag{3}
\]

where \( Q_o = Q_d - u \).

Next, we next derive and analyze the manufacturer’s and the distributor’s decisions.

2.1 Manufacturer’s Decision

Because the manufacturer does not know the distributor’s ordering policy, the manufacturer develops a prior function \( \hat{Q}(P_m) \) to forecast the distributor’s order quantity, where \( \hat{Q}(P_m) \) satisfies \( \hat{Q}'(P_m) < 0 \) and \( \hat{Q}''(P_m) > 0 \) (i.e., \( \hat{Q}(P_m) \) is decreasing convex in \( P_m \)). From (2), the manufacturer’s perceived expected profit function will be

\[
E[\Pi_m(P_m)] = \begin{cases} 
\int_0^{\hat{Q}(P_m)} (P_m - c_m u - P_o \hat{Q}(P_m) - u) dQ_d & \text{if } \hat{Q}(P_m) > u \\
\int_{\hat{Q}(P_m)}^\infty (P_m - c_m \hat{Q} \hat{Q}(P_m) - c_o \hat{Q}(P_m)) - P_o \hat{Q}(P_m) dQ_d & \text{otherwise.} 
\end{cases} \tag{4}
\]

The manufacturer will set his unit sales price equal to \( P_m^* \) in order to maximize (4), and will provide his unit sales price to the distributor. Lemma 1 shows that the manufacturer’s perceived profit function is unimodal.

**Lemma 1** The manufacturer’s perceived expected profit function is unimodal.

**Proof.** The derivation is shown in Appendix A.

2.2. Distributor’s Decision

After receiving the manufacturer’s quotation for the unit sales price \( P_m \), the distributor determines his unit sales price \( P_d \) and ordering quantity \( Q_d \) to maximize his expected profit function. From (1), the distributor’s expected profit function is given by

\[
E[\Pi_d(Q_d, P_d)] = \int_0^{Q_d} (P_d x - P_m Q_d) \cdot f(x | p_d) dx + \int_{Q_d}^\infty (P_d - P_m) Q_d \cdot f(x | p_d) dx. \tag{5}
\]
By substituting \( Q_d = z m(p_d) \) into (5), we transform \( E[\Pi_d(Q_d, p_d)] \), which is a two-variable function, into a one-variable function:

\[
E[\Pi_d(p_d)] = \int_0^{z m(p_d)} (p_d \times - p_m \cdot z \cdot m(p_d)) \cdot f(x \mid p_d) \, dx + \int_0^\infty (p_d - p_m) \cdot z \cdot m(p_d) \cdot f(x \mid p_d) \, dx.
\]

The distributor’s unit sales price can be determined by maximizing \( E[\Pi_d(p_d)] \). We prove in Lemma 2 that \( E[\Pi_d(p_d)] \) has a unique maximizer.

**Lemma 2** The distributor’s expected profit function \( E[\Pi_d(p_d)] \) has a unique maximizer.

**Proof.** The derivation is shown in Appendix B.

The distributor will set his unit sales price \( p_d^* \) to be \( p_d^* = \frac{k \cdot z \cdot p_m \cdot e}{k - 1} \).

The distributor will order \( Q_d^* \) quantities from the manufacturer, where \( Q_d^* \) is

\[
Q_d^* = z D \left( \frac{k \cdot z \cdot p_m \cdot e}{k - 1} \right)^{-k}.
\]

and the distributor’s maximized expected profit function is

\[
E[\Pi_d(p_d^*)] = a \left( \frac{k \cdot z \cdot p_m \cdot e}{k - 1} \right)^{-k} \left( \frac{k \cdot z \cdot p_m \cdot e}{k - 1} - p_m \cdot z \cdot e \right),
\]

where

\[
a = D \int_0^1 f(y) \, dy, \quad a = 1 / \int_0^1 f(y) \, dy, \quad a = x / m(p_d).
\]

The distributor’s sales price, which is defined in (B. 1), depends on his attitude towards risk \( z \). Thus, we desire to know how the distributor’s attitude toward risk affects the optimal unit sales price \( p_d^* \). Differentiating \( p_d^* \) with respect to \( z \) gives

\[
\frac{dp_d^*}{dz} = \frac{k \cdot p_m \cdot e}{(k - 1)} > 0.
\]

That is, when the value of \( z \) decreases, the distributor’s optimal unit sales price will decrease, implying that a risk-averse distributor would like to charge a lower price. In other words, a conservative distributor hopes to sell as much as possible the order quantity \( Q_d(p_m) \) with a lower unit sales price.

In the absence of coordination, the manufacturer’s and the OEM’s profits will be \( \Pi_m(Q_m) \) and \( \Pi_o(Q_o) \), respectively, where \( Q_m = [Q_o - u]^+ \).

We can observe that each supply chain member’s decision is determined by maximizing his own expected profit function, and thus we propose a mechanism to coordinate the members’ decisions in order to improve the total supply chain profit. More discussions on coordination mechanisms will be presented in Section 3.

### 3. Supply Chain Coordination Model

In this section, we develop a mechanism to coordinate the manufacturer’s and the distributor’s ordering and pricing policies in order to stimulate more market demand and to increase the total supply chain profit. We consider two coordination scenarios: (1) Both the manufacturer and the distributor participate in coordination, but the OEM does not; (2) All the supply chain members participate in coordination. These two coordination scenarios will improve our understanding of the effects of different degrees of supply chain coordination.

In Scenario 1, the manufacturer provides his unit production cost \( c_m \) as his unit sales price to the distributor in order to lower the distributor’s unit sales price, but the OEM does not provide his unit production cost \( c_o \) as his unit sales price for the manufacturer. In Scenario 2, the manufacturer and the OEM, respectively, provide the unit production costs \( c_m \) and \( c_o \) as the unit sales prices for the distributor and the manufacturer. We assume that the OEM’s unit production cost \( c_o \) is larger than the manufacturer’s unit production cost \( c_m \). Thus, the manufacturer in Scenario 2 will produce the quantity within his capacity, and outsource insufficient capacity to the OEM. The members who provide their unit production costs as the unit sales prices will have no profit of selling products; thus, the distributor will share a portion of his profit with other members who participate in the coordination. The joint expected profit functions and the coordinated decisions in Scenario 1 and Scenario 2 will be analyzed in Section 3.1. In Section 3.2, we analyze the optimal ordering and pricing decisions in the absence and presence of coordination.

#### 3.1 Coordinated Decisions

We derive the optimal coordinated ordering and pricing policies for Scenario 1 and Scenario 2 in sequence as follows.

##### 3.1.1 Scenario 1

In Scenario 1, the manufacturer’s unit sales price \( p_m \) is set as his unit production cost \( c_m \). The manufacturer outsources insufficient capacity to the OEM with the unit sales price \( p_o \). While determining the distribu-
tor’s sales policies from the expected joint profit function \( E[\Pi_j(p_s)] \), the distributor is considered work with the manufacturer in a firm (i.e., they are in a centralized supply chain). When \( Q_d > u \), the case we focus in this paper, the expected joint profit function is given by the following:

\[
E[\Pi_j(p_s)] = \int_0^{Q_d} (p_d x - c_m u - p_m (Q_d - u)) f(x | p_s) dx
\]

\[
+ \int_0^{\infty} (p_d Q_d - c_m u - p_m (Q_d - u)) f(x | p_s) dx,
\]

where \( Q_d = z D(p_d)^{\frac{1}{k}} \). When \( Q_d \leq u \), the expected joint profit function can be simplified as

\[
E[\Pi_j(p_s)] = \int_0^{Q_d} (p_d x - c_m u) f(x | p_s) dx
\]

\[
+ \int_0^{\infty} (p_d Q_d - c_m u) f(x | p_s) dx,
\]

which is similar to (Weng, 1999), which is not in the research scope of this study. By maximizing (6), the optimal coordinated ordering and pricing policies in the presence of coordination can be derived as follows.

Lemma 3 The optimal coordinated ordering and pricing policies in Scenario 1 are

\[ p_d^* = \frac{k z p_m e}{k - 1}, \]

and

\[ Q_d^* = zD \left( \frac{k z p_m e}{k - 1} \right)^{\frac{1}{k}}, \]

where \( k > 1 \) and \( e = 1/(\int_0^{\infty} f(y) dy) \).

Proof. The derivation follows from Lemma 2.

The maximized joint expected profit is

\[
E[\Pi_j(p_s^*)] = a \left( \frac{k z p_m e}{k - 1} \right)^{1/(k z p_m e)} \left( \frac{k z p_m e}{k - 1} \right)
\]

\[
- p_m z e - c_m u + p_m u,
\]

where

\[ a = D \int_0^{\infty} f_j(y) dy, \quad e = 1/(\int_0^{\infty} f_j(y) dy), \quad y = x/m(p_s). \]

The manufacturer’s outsourcing quantity \( Q_m \) is equal to \( Q_d^* - u \), and the OEM’s profit in Scenario 1 can be derived by substituting \( Q_m = Q_d^* - u \) into (3).

3.1.2 Scenario 2

In Scenario 2, both the OEM and the manufacturer participate in the coordination, and set their unit production costs as the unit sales prices. When \( Q_d > u \), the joint expected profit function can be derived from (6) by replacing \( p_m \) with \( c_m \). When \( Q_d \leq u \), the joint expected profit function is the same as (7), which is independent of the OEM’s unit sales price.

The coordinated optimal ordering and pricing policies in Scenario 2 are presented in the following lemma.

Lemma 4 The coordinated optimal ordering and pricing policies in Scenario 2 are

\[ p_d^* = \frac{k z c_m e}{k - 1}, \]

and

\[ Q_d^* = zD \left( \frac{k z c_m e}{k - 1} \right)^{\frac{1}{k}}, \]

where \( k > 1 \) and \( e = 1/(\int_0^{\infty} f_j(y) dy) \).

Proof. The derivation follows from Lemma 2.

The maximized joint expected profit is

\[
E[\Pi_j(p_s^*)] = a \left( \frac{k z c_m e}{k - 1} \right)^{1/(k z c_m e)} \left( \frac{k z c_m e}{k - 1} \right)
\]

\[
- c_m z e - c_m u + c_m u,
\]

where \( a = D \int_0^{\infty} f_j(y) dy, \quad e = 1/(\int_0^{\infty} f_j(y) dy), \quad y = x/m(p_s). \)

3.2 Analysis of the Optimal Decisions

Because \( p_m > p_m > c_m \), we obtain that

\[ \frac{k z p_m e}{k - 1} > \frac{k z p_m e}{k - 1} > \frac{k z c_m e}{k - 1} \]

and that \( Q_d^* < Q_d^* < Q_d^* \). Thus, the distributor order quantity \( Q_d \) in the presence of coordination is larger than that in the absence of coordination. That is Scenario 2 with a higher degree of coordination can stimulate the largest order quantity and the lowest sales price.

The total expected profits in the absence of coordination, \( E[\Pi_j(p_s^*)] \), and the total expected profits in Scenario 1, \( E[\Pi_j(p_s^*)] \), can be derived as follows:

\[
E[\Pi_j(p_s^*)] = a \left( \frac{k z p_m e}{k - 1} \right)^{1/(k z p_m e)} \left( \frac{k z p_m e}{k - 1} \right)
\]

\[
- c_m z e - c_m u + p_m u,
\]

and

\[
E[\Pi_j(p_s^*)] = a \left( \frac{k z p_m e}{k - 1} \right)^{1/(k z p_m e)} \left( \frac{k z p_m e}{k - 1} \right)
\]

\[
- c_m z e - c_m u + p_m u,
\]

where
\[ a = D \int_{0}^{y} f_{x}(y)dy, \quad c = 1/ \int_{0}^{y} f_{x}(y)dy, \quad \text{and} \quad y = x/m(p_{d}). \]

Because the total expected profit, \( E[\Pi_{i}^{2}(p_{s}')] \), in Scenario 2 is equal to (9), and then we observe that if \( p_{w} > p_{o} > c_{o} \), the relationship follows

\[ E[\Pi_{i}^{2}(p_{s}')] < E[\Pi_{i}^{2}(p_{s}')] < E[\Pi_{i}^{2}(p_{s}')] \).

Hence, the coordination can improve the total supply chain profit, and higher degree of coordination can increase more the total supply chain expected profit.

4. Numerical Study

We present a numerical example to illustrate the performance of coordination. We consider the pdf of the random market demand to be

\[ f(x | p_{d}) = \frac{1}{m(p_{d})} e^{m(p_{d})x}, \quad f(x | p_{d}) = \frac{1}{m(p_{d})} e^{m(p_{d})x}, \]

which is a special case of the generalized phase-type distribution which can be used to represent any non-negative random variable.

To evaluate the performance of coordination, we define a measure \( \Delta \) as follows:

\[ \Delta = \left( \frac{\Pi_{j} + \Pi_{k}}{\Pi_{j} + \Pi_{k} + \Pi_{j}} \right) \times 100\% . \]

The coordination would be beneficial if \( \Delta \geq 0 \) is held. In this numerical study, we consider the following parameter setting: \( p_{o} = 2.5, c_{o} = 2, p_{w} = 3 \times c_{w}, c_{w} = 1, u = 500, D = 200000, k = 1.5, \) and \( z = 1. \) Then, we vary the value of each parameter to show how it influences the coordination performance, while holding the other parameters.

Both Figure 1(a) and (b) show that higher degrees of attitude toward risk result in higher performance of coordination. A lower value of \( z \) means that the distributor’s order quantity is not so sensitive to \( p_{s} \), leading to less order quantities in the absence and presence of coordination.

The decrease of \( z \) reduces the difference between \( Q_{d}^{*} \) and \( Q_{e}^{*} \), and decreases the performance of coordination. Therefore, the distributor who is a conservative decision maker will have lower coordination performance. Comparing Figures 1(a) and (b), we find that higher degree of integration leads to higher performance.

![Fig. 1](image1.png) (a) The measure \( \Delta \) versus \( z \) in Scenario 1; (b) The measure \( \Delta \) versus \( z \) in Scenario 2.

![Fig. 2](image2.png) (a) The measure \( \Delta \) versus \( k \) in Scenario 1; (b) The measure \( \Delta \) versus \( k \) in Scenario 2.
Table 1. Parametric Value

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The manufacturer’s unit production cost $c_m$</td>
<td>1, 1.1, 1.2, · · · , 2</td>
</tr>
<tr>
<td>The OEM’s unit production cost $c_o$</td>
<td>0, 0.2, 0.4, · · · , 2.4</td>
</tr>
<tr>
<td>The OEM’s unit sales price $p_o$</td>
<td>2, 3, 4, · · · , 6</td>
</tr>
<tr>
<td>The constant price elasticity $k$</td>
<td>1.2, 1.4, 1.6, · · · , 2</td>
</tr>
<tr>
<td>The distributor’s attitude toward risk $z$</td>
<td>0.4, 0.6, 0.8, · · · , 4</td>
</tr>
</tbody>
</table>

Figures 2(a) and (b) illustrate that the performance increases as $k$ increases. As mentioned in Section 3.2 that larger degrees of coordination will reduce the distributor’s unit sales price. When the market demand is more price-sensitive (i.e., $k$ is larger), the decrease of $p_d$ stimulates larger market demand. Thus, the performance of coordination will increase as $k$ increases, and higher degree of coordination will have higher performance.

Figure 3(a) presents that $\Delta$ decreases in $p_o$ in Scenario 1, which since the manufacturer’s profit decreases in $p_o$, and the coordinated model is more sensitive to $p_o$ than the basic model. In Scenario 2, the OEM’s provides his unit production cost as the unit sales price, and thus $\Delta$ is independent of $p_o$, as shown in Figure 3(b).

That $\Delta$ decreases in $c_o$, as depicted in Figures 4 (a) and (b). We can observe the OEM’s profit decreases as $c_o$ increases. Because $Q^*_o > Q^*_d$, the coordinated models are more sensitive to the decrease of the OEM profit than the uncoordinated model.

5. Summary

This paper proposes the coordination mechanism, and discusses how coordination influences the performance. The results of our study indicate that the coordination mechanism can stimulate more demand and improve all the supply chain members’ profits. The supply chain in the presence of coordination is more sensitive to the variation of the parameters than the scenario in the absence of coordination, and the reason may be due to the fact that all members share their information in the presence of coordination. Higher degrees of integration will lead the distributor to set lower unit sales price, and to order larger quantity. The performance of coordination will be improved when the coordination is present, and higher degrees of coordination will have higher coordination performance. Our coordination mechanism is more suitable for the market which is more-price sensitive and for the distributor who is a risk-toward decision maker.
References


Appendix

Appendix A: Proof of Lemma 1

Proof. The first and second derivatives of \( E[\Pi_m(p_m)] \) with respect to \( p_m \), respectively, are

\[
E[\Pi_m(p_m)] = \hat{Q}(p_m) + (p_m - p_m)\hat{Q}'(p_m)
\]

and

\[
E''[\Pi_m(p_m)] = 2\hat{Q}(p_m) + (p_m - p_m)\hat{Q}''(p_m).
\]

We can verify that \( E'[\Pi_m(p_m)] = 0 \) has a unique positive root \( p_m^* \), which is

\[
p_m^* = p_m - \frac{\hat{Q}(p_m)}{\hat{Q}'(p_m)} > 0.
\]

Since \( dE[\Pi_m(p_m)]/dp_m \rightarrow +\infty \) as \( p_m \rightarrow 0 \) and \( dE[\Pi_m(p_m)]/dp_m \rightarrow -\infty \) as \( p_m \rightarrow \infty \),

\[
dE[\Pi_m(p_m)]/dp_m > 0 \forall p \in [0, p_m^*],
\]

and

\[
dE[\Pi_m(p_m)]/dp_m < 0 \forall p \in (p_m^*, \infty].
\]

Hence, the solution \( p_m^* \) obtained from \( dE[\Pi_m(p_m)]/dp_m = 0 \) is the one that maximizes the manufacturer’s predicted expected profit function. By letting \( E'[\Pi_m(p_m)] = 0 \), we obtain that

\[
p_m = \frac{-\hat{Q}(p_m) + \hat{Q}'(p_m)}{\hat{Q}''(p_m)} > 0.
\]

Since \( d^2E[\Pi_m(p_m)]/dp_m^2 \rightarrow -\infty \) as \( p_m \rightarrow 0 \),
and \( d^2 E[\Pi_m(p_n)]/d(p_n)^2 \to +0 \) as \( p_n \to \infty \), we have
\[
d^2 E[\Pi_m(p_n)]/d(p_n)^2 < 0 \quad \forall \ p \in [0, \frac{-2\hat{q}'(p_n)}{\hat{q}'(p_n)} + p_n\hat{q}'(p_n)],
\]
and
\[
d^2 E[\Pi_m(p_n)]/d(p_n)^2 > 0 \quad \forall \ p \in (-\frac{2\hat{q}'(p_n)}{\hat{q}'(p_n)} + p_n\hat{q}'(p_n), \infty).
\]
Therefore, \( E[\Pi_m(p_n)] \) is unimodal and has a unique maximizer \( p_n^* \).

Appendix B: Proof of Lemma 2

**Proof.** The distributor’s expected profit function is
\[
E[\Pi_d(p_d)] = \int_{x=m(p_d)}^{\infty} [p_d x - p_n(z \cdot m(p_n))] \cdot f(x | p_d)dx + \int_{x=m(p_d)}^{\infty} (p_d - p_n)z \cdot m(p_d) \cdot f(x | p_d)dx.
\]
We transform \( E[\Pi_d(p_d)] \) into
\[
E[\Pi_d(p_d)] = P \int_{x=m(p_d)}^{\infty} [1 - F_x(x)]dx - p_n z \cdot m(p_n)
\]
\[
= m(p_n) \cdot p_d \int_{y=0}^{\infty} [f_y(y)]dy - p_n z
\]
\[
= D(p_d)^{-k} \cdot p_d \cdot \frac{p_n z}{\int_{y=0}^{\infty} [f_y(y)]dy} \int_{y=0}^{\infty} [f_y(y)]dy
\]
\[
= a(p_d)^{-k} \cdot (p_d - p_n z \cdot c),
\]
where
\[
a = D \int_{0}^{\hat{y}} f_y(y)dy, \ \hat{y} = 1/\int_{0}^{\hat{y}} f_y(y)dy, \ \text{and} \ \hat{y} = x / m(p_d).
\]
The first derivative of \( E[\Pi_d(p_d)] \) with respect to \( p_d \) is
\[
E'[\Pi_d(p_d)] = a(p_d)^{-k} \cdot (p_d - k p_d + k p_n z \cdot c).
\]
Solving \( E'[\Pi_d(p_d)] = 0 \) gives a critical point of \( p_d \)
\[
p_d^* = \frac{c z p_n}{k - 1}.
\]
When \( p_d \to 0 \), \( E'[\Pi_d(p_d)] \to \infty \); and when \( p_d \to \infty \), \( E'[\Pi_d(p_d)] \to -0 \). Thus, we have
\[
E'[\Pi_d(p_d)] > 0 \quad \text{when} \quad p_d < p_d^*
\]
and
\[
E'[\Pi_d(p_d)] < 0 \quad \text{when} \quad p_d > p_d^*.
\]
The second derivative of \( DP(p_d) \) is
\[
E''[\Pi_d(p_d)] = a k (p_d)^{-k} [(k - 1)p - (k + 1)p_n z \cdot c].
\]
And, we can find that \( E''[\Pi_d(p_d)] \) is an increasing function of \( p_d \). By letting \( DP''(p_d) = 0 \), we obtain that
\[
p_d = \frac{(k + 1)p_n z \cdot c}{(k - 1)} > 0.
\]
Since \( E''[\Pi_d(p_d)] \to -\infty \) as \( p_d \to 0 \), and \( E''[\Pi_d(p_d)] \to +0 \) as \( p_d \to \infty \);
\[
d E''[\Pi_d(p_d)] < 0 \forall p \in [0, \frac{(k + 1)p_n z \cdot c}{(k - 1)}],
\]
and
\[
E''[\Pi_d(p_d)] > 0 \forall p \in (\frac{(k + 1)p_n z \cdot c}{(k - 1)}, \infty].
\]
Therefore, \( E[\Pi_d(p_d)] \) has the unique maximizer \( p_d^* \).