An Approach to Condition the Transition Matrix on Credit Cycle: An Empirical Investigation of Bank Loans in Taiwan

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Abstract

Credit risk models for credit rating transitions are important ingredients for credit risk management. This paper propose a formal model, conditional Markov chain model, for gauging credit risk and apply it to Taiwan’s bank loans from 1998-2003. The model’s central feature is to incorporate credit cycle and time-varying risk premium into transition matrices. There are three main contributions. First, we apply the methodology to bank loans in Taiwan, which is more elaborate than previous studies. Second, the empirical results show that credit cycle and risk premium have significant influence on default probabilities. That is, if we ignore credit cycle and risk premium, default probabilities may be underestimated. Third, the estimation procedures for assessing the credit risk of financial institutions are easy to follow and implement. On the whole, the proposed model can provide more reliable estimated results for credit risk of bank loans.

Keywords: Default probability; Bank loan; Credit cycle; Risk premium

1. Introduction

Over a decade ago, the Basel Committee on Banking Supervision (“The Committee”) produced guidelines for determining bank regulatory capital. The objective of this accord was to level the global playing field for financial institutions and protect all risks in the financial system. In 1988, the Committee issued “the International Convergence of Capital Standard” (or “Capital Accord of 1988”), which established regulations regarding the amount of capital that banks should hold against the credit risk. Furthermore, the treatment of market and the operational risk were incorporated in 1996 and 2001, respectively. The final version of the Accord was published in 2004 and will be implemented after 2006.1

Most statistics showed a significant increase in the credit risk, such as Japan, Korea and United States of America etc.2 In the past, the Committee approach has been described as a “one size fit all”. Now, the Committee allows banks using “internal models” rather than the alternative regulatory (“standardized”) model to assess their credit risks. As a result, most new models and technologies were emerged applying to analyze credit risk. An accurate credit risk analysis becomes even more important today.

Although credit risk models have been developed rapidly such as Jarrow, Lando and Turnbull (1997); Kijima and Komoribayashi (1998) and Wei (2003), they do not consider the credit risk of the bank loans. For bank industry, credit risk is associated with the quality of individual assets (or loans) and the likelihood of default. Credit risk is the potential variation in net income and market value of assets resulting from this nonpayment or delayed payment. Different types of loans have different default probabilities. On the other hand, changes in economic conditions and a bank’s operating environment alter the cash flow of their assets, such as loans. Therefore, it is extremely difficult to assess credit risk of bank loans.

The purpose of this paper is to provide an effective model to measure the credit risk. The model is more elaborate than previous studies such as Jarrow, Lando and Turnbull (1997) and Wei (2003) that ignore the

1 E-mail: lotus-lynn@nuu.edu.tw
2 For Japan, there are Daiwa Bank failure and Yamaichi Securities failure in 1995 and 1997, respectively. Long Term Credit Bank of Japan (LTCB) was one of the two biggest bank failures of the twentieth century, with Nippon Credit Bank being the other. For Korea, the Korean Development Bank is a famous case. There are many factors that cause banks to default; however, credit risk is number one.
credit cycle and time-varying risk premium. In this paper, we contribute to the literature in the following aspect. First, the model incorporates credit cycle in transition matrices. Although Lu and Kuo (2006) have applied Markov chain model to bank loans, they do not consider the credit cycle, which is an important factor. Therefore, this paper serves as one of the first studies to adopt a conditional Markov chain model for assessing the credit risk of bank loans, which has never been discussed in previous studies. In addition, we compare the difference between the observed and fitted transition matrices. The observed transition matrix is calculated from the original Markov chain model that doesn’t consider credit cycle. On the other hand, we incorporate the credit cycle with the transition matrix, so-called fitted transition matrix.

Second, when gauging credit risk of bank loans, risk premium plays a crucial role. However, previous researches handle the risk premium as time-invariant (Jarrow, Lando and Turnbull, 1997; Wei, 2003). In fact, the risk premium is not time-invariant but is actually always time-variant (Kijima and Komoribayashi, 1998; Lu and Kuo, 2006). Therefore, we will also relax assumptions of previous researches by incorporating time-variant risk premium, which is more elaborate than previous researches. Third, the estimated procedures are easy to follow and implement. The model can apply to value the credit risk of other financial institutions. On the whole, we expect that the proposed model not only provides an effective credit risk review for financial institutions but also help to face the Basel Capital Accord.

This paper is organized as follows. Section 1 provides motivation. Section 2 reviews literature concerning models of credit risk. Section 3 presents the formal methodology, and Section 4 describes the sample data used in this paper. Section 5 shows the main empirical results. Finally, Section 6 includes a discussion of our findings with a conclusion.

2. Literature Review

Over the last few years, credit risk modeling and credit derivatives valuation have received tremendous attention worldwide. These credit risk models can be grouped into two main categories: the structural-form model and the reduced-form model. One important difference between these two categories of models is the implicit assumption they make about managerial decisions regarding their capital structure. The structural-form model is also called the asset value model for assessing credit risk, typically of a corporation’s debt. It was based on the principle of pricing option in Black and Scholes (1973) and a more detailed model developed by Merton (1974).

Furthermore, the basic Merton model has subsequently been extended by removing one or more of his assumptions. For instance, Black and Cox (1976) and Kim, Ramaswamy and Sundaresan (1989) suggest that capital structure is explicitly considered and default occurs if the value of total assets is lower than the value of liabilities. Brennan and Schwartz (1978) and Longstaff and Schwartz (1995) investigate the stochastic interest rate correlated with the firm process. Leland (1994) endogenizes the bankruptcy while accounting for taxes and bankruptcy costs.

Over the last few years, KMV Corporation has developed a credit risk methodology to assess default probabilities and the loss distribution related to default risks. KMV derives the default probability for each obligor based on Merton model. KMV best applies to publicly traded companies for which the equity’s value is market determined. The information contained in the firm’s stock price and balance sheet can then be translated into an implied risk of default. Thus, the default probability is a function of the firm’s capital structure, the volatility of the asset returns and the current asset value. Consequently, structural-form models rely on the balance sheet of the borrower and the bankruptcy code in order to derive the probability of default.

In spite of the extensions of Merton’s original framework, these models still suffer some drawbacks. First, since the firm’s value is not a tradable asset, nor is it easily observable, the parameters of the structural-form model are difficult to estimate consistently. Second, the inclusion of some frictions like tax shields and liquidation costs would break the last rule. Third, corporate bonds undergo credit downgrades before they actually undergo default, but structural-form models cannot incorporate these credit-rating changes.

Reduced-form models attempt to overcome these shortcomings of structural-form models, such as Jarrow and Turnbull (1995), Duffie and Singleton (1997), Jarrow, Lando and Turnbull (1997) and Lando (1998). Unlike structural-form models, reduced-form model

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3 Despite there have been extensive literatures on Markov chain model such as Hamilton (1989), Turn, Startz and Nelson (1989), Engle and Hamilton (1990), Kaminsky and Peruga (1990), Li and Lin (2000), Chae and Lee (2005) and Chang, Tu and Tsai (2005), they also don’t consider the credit cycle. Chae and Lee (2005) propose a method for stationary probabilities of the embedded Markov chain of a class of G/M/1 queueing systems. Chang, Tu and Tsai (2005) suggest a numerical method that relies on the Markov chain approximation to compute the optimal strategies. Li and Lin (2000) adopt Markov chain model to examine the return variability for major East Asian market indices.

4 There are other approaches to measure credit risk such as Lee and Chang (1998) and Lu and Kuo (2005). Lee and Chang (1998) investigate the advanced officers of credit rating department on the bank by questionnaire that used to establish the credit grading standard for middle and small business in Taiwan. On the other hand, Lu and Kuo (2005) assess the credit risk of a Taiwanese bills finance company by a marketed-based approach.

5 KMV Corporation is a firm specialized in credit risk analysis.
make no assumptions at all about the capital structure of the borrowers. The calibration of this probability of default is made with respect to the data of ratings agencies. The original Jarrow and Turnbull (1995) model, which was perhaps the first reduced-form model to experience widespread commercial acceptance, was worked out through the use of matrices of historical transition probabilities from the original ratings and recovery values at each terminal state. In contrast, reduced-form models can extract credit risks from the actual market data and are not dependent on asset value and leverage. Therefore, parameters that are related to the firm’s value need not be estimated in implementation.

On the other hand, the current proposed industry also sponsored methodologies for measuring credit risk, such as CreditMetrics, KMV, CreditRisk+ and CreditPortfolio View. CreditMetrics was introduced in 1997 by J. P. Morgan and its co-sponsors (Bank of America, Union Bank of Switzerland, Swiss Bank Corporation, BZW, Deutsche Morgan Grenfell, etc.) as a value at risk framework to apply to the valuation and risk of nontradable assets such as loans and privately placed bonds. The approach of CreditMetric is based on credit rating transition analysis, i.e. the probability of moving from one credit rating to another, including default, within a given horizon, which is often taken arbitrarily as 1 year. CreditMetrics’ approach applies primarily to bonds and loans and it can be easily extended to any type of financial claims.

KMV’s methodology, which is described previously, differs somewhat from CreditMetrics as it relies upon the “Expected Default Frequency”, or EDF, for each issuer, rather than upon the average historical transition frequencies produced by the rating agencies, for each credit rating. That is, KMV does not use Moody’s or Standard & Poor’s statistical data to assign a default probability which only depends on the rating of the obligor.

CreditRisk+ is a model developed by Credit Suisse Financial Products (CSFP) at the end of 1997. CreditRisk+ assumes that default for individual bonds, or loans, follows a Poisson process and attempts to estimate the expected loss of loans and the distribution of those losses, with a focus on calculating the required capital reserves of financial institutions.

CreditPortfolio View was proposed McKinsey, which is a consulting firm. CreditPortfolio View is a multi-factor model which is used to simulate the joint conditional distribution of default and migration probabilities for various rating groups in different industries. The default probability for CreditPortfolio View is condition on the value macroeconomic factors like the unemployment rate, the rate of growth in GDP, the long-term interest rates, foreign exchange rates, government expenditures and the aggregate saving rate.

From a credit risk modeling perspective, variation in transition matrices attribute to credit cycle is potentially important. Belkin, Suchower and Forest (1998) employ a parameter to measure the credit cycle, which means the values of both default rates and end-of-period risk ratings are not predicted by the initial mix of credit grades. Kim (1999) builds a credit cycle index, which is similar to that of Belkin, Suchower and Forest (1998). As well-known, credit ratings changes play a crucial role in many credit risk models.

Wilson (1997) has shown that transition probabilities change over time as the state of the economy evolves. In this approach, default probabilities are a function of macrovariables such as unemployment, interest rate, the growth rate, government expenses, and foreign exchange rates. He uses macrovariables to drive the credit cycle. Unfortunately, variables as proxies for macrovariables are seldom completely satisfactory. Whereas some of the contributions in the literature introduce observed macrovariables to capture co-variation in default intensities between firms and economy environment, an alternative approach is to estimate the common components of default risk directly from data. Unlike Wilson (1997), we will estimate conditional default probabilities directly from the data. The Basel Committee on Banking Supervision also emphasizes the importance of the credit cycle, which may improve the accurate assessments of credit risk. Therefore, we will estimate credit cycle to construct a fitted transition matrix that is first contribution in this paper.

On the other hand, although Wilson (1997) attempt to incorporate the credit cycle into transition matrices, they don’t consider the risk premium. Recently, Jarrow, Lando and Turnbull’s (1997) model matches the Committee’s opinion reasonably well, and represents a major step forward in credit risk modeling. The model of Jarrow, Lando and Turnbull (1997) is based on the risk-neutral probability valuation model, also called the Martingale approach to pricing of securities, which derives a risk premium for the dynamic credit rating process from the Markov chain process and then estimates the default probability by transition matrix.

The risk-neutral probabilities use to value risky assets has been in the financial literature at least as far back as Arrow (1953) and has been subsequently developed by Harrison and Kreps (1979), Harrison and Pliska (1981), and Kreps (1982). In finance, it has been traditional to value risky assets by discounting cash flows. Risk-neutral probabilities can be used in setting the required spread or risk premium on a loan. The risk-neutral valuation framework provides valuable
tools for both default prediction and loan valuation. Compared to history-based (or data-based) transition probabilities, the risk-neutral model gives a forward-looking prediction of default. In general, the risk-neutral default probability will exceed the history-based transition default probability over some horizon because it contains a risk premium reflecting the unexpected probability of default.

Furthermore, Wei (2003) proposes the multi-factor, Markov chain model that considers the credit cycle and risk premium on transition matrix. He allows the transition matrix to evolve according to the credit cycle. Although his model is more general than previous studies, he assumes the risk premium is kept constant over time. Kijima and Komoribayashi (1998) and Lu and Kuo (2006) propose a procedure to estimate the risk premium, which is not time-invariant but is actually always time-variant. As a result, there are two important factors, the credit cycle and the risk premium, for assessing the credit risk. The paper relaxes the assumptions of previous researches by incorporating the credit cycle and time-variant risk premium into Markov chain model for assessing the credit risk of bank loans, which is more elaborated than previous studies. Consequently, we expect that this work can improve the accuracy of the assessment of credit risk and be helpful for all banks with an internal rating system, including Taiwan’s banking industry.

3. Model Specification
3.1 Observed Transition Matrix

Credit ratings of firms are published in a timely manner by rating agencies, such as Standard & Poor’s or Moody’s. The importance of credit ratings has increased significantly with the introduction of the Basel II. They provide investors with invaluable information to assess the firms’ abilities to meet their debt obligations. If a company’s credit quality has improved or deteriorated significantly over time, such a review will prompt the agency to raise or lower its rating.

It is obvious that the present rating of an obligor is a predictor for its rating in the nearest future. A cardinal feature of any credit ratings is past and present ratings influencing the evolution. Therefore, the Markov chain is a stochastic process of this kind. In recent years, it has become common to use a Markov chain model to describe the dynamics of firm credit ratings such as Jarrow and Turnbull (1995) and Jarrow, Lando and Turnbull (1997).

To be more specific, let \( x_t \) represent the credit rating at time \( t \) of a bank’s borrower. We assume that \( x = \{x_t, t = 0, 1, 2, \ldots \} \) is a time-homogeneous Markov chain on the state space \( S = \{1, 2, \ldots, C, C+1\} \), where state 1 represents the highest credit class, state 2 the second highest, ..., state \( C \) the lowest credit class, state \( C+1 \) designates the default. It is usually assumed for the sake of simplicity that the default state \( C+1 \) is absorbing. Furthermore, let \( f_{ij} = P(x_{t+1} = j | x_t = i) \), \( i, j \in S, t = 0, 1, 2, \ldots \) denote the probability of state \( i \) transiting to state \( j \) through the actual probability measure. That is, \( f_{ij} \) and \( P \) represent the one-step transition probability and actual probability measure, respectively. Then, the discrete time and the regime-switching of credit class \( i \) transiting to credit class \( j \) can be represented by a time-homogeneous transition matrix according to the following, and is called as the observed transition matrix.

\[
F = \begin{bmatrix}
    f_{11} & f_{12} & \cdots & f_{1C} & f_{1,C+1} \\
    f_{21} & f_{22} & \cdots & f_{2C} & f_{2,C+1} \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    f_{C1} & f_{C2} & \cdots & f_{CC} & f_{C,C+1} \\
    0 & 0 & \cdots & 0 & 1
\end{bmatrix}
\]

where \( f_{ij} \geq 0 \quad \forall i, j \) and \( \sum_{j=1}^{C} f_{ij} = 1 \quad \forall i \). The submatrix \( A_{(C \times C)} \) is defined on non-absorbing states \( \hat{S} = \{1, 2, \ldots, C\} \). The components of submatrix \( A \) denote the regime-switching of credit classes for the bank’s borrower, however, it excludes default state \( C+1 \). \( D_{(C \times 1)} \) is the column vector with components \( f_{i,C+1} \), which represent the transition probability of banks’ borrowers for any credit class, i.e., \( i = 1, 2, \ldots, C \), switching to a default class, i.e., \( j = C+1 \). We assume for the sake of simplicity that bankruptcy (state \( C+1 \)) is an absorbing state, so that \( \begin{bmatrix} O_{(C \times C)} \end{bmatrix} \) is the zero column vector giving a transition probability from the default state from initial to the final time. Once the process enters the default state, it would never return to credit class state, so that \( f_{C+1,C+1} = 1 \). In such a case, we would say that the default state \( C+1 \) is an absorbing state.

3.2 Fitted Transition Matrix

For considering the credit cycle, there are some procedures to calculate the fitted transition matrix. The first step is to devise a mapping through which the transition probability can be translated into credit scores. The paper employs the normal distribution which is easy to calculate. Since the summation of each row in a transition matrix is always equal to 1, we can invert the cumulative normal distribution function starting from the default function as in Belkin, Suchower and Forest (1998), Kim (1999) and Wei (2003). Therefore, the observed transition matrix, ma-
Matrix (2) is \((C \times C)\) because there is no need to covert the row for the absorbing state, i.e., default state. If we convert matrix (2) into a probability transition, then we get a transition matrix as in matrix (1). As in Belkin, Suchower and Forest (1998), we decompose \(Y\) of time-\(t\) into two factors:

\[
Y_t = \alpha L_t + \sqrt{1-\alpha^2} \varepsilon_t
\]

(3)

where \(L_t\) is the systematic component shared by all borrowers, meaning "credit cycle". The credit cycle will be positive in good year, which implies that for each initial credit rating, a rate lower than average default and higher than the average of upgrades to downgrades. On the other hand, the credit cycle will be negative in bad year. In any year, the observed transition matrix as shown above will deviate from the norm, that is \(L_t = 0\). Here \(\varepsilon_t\) is a non-systematic, and the idiosyncratic factor is unique for a borrower. We also assume that \(L_t\) and \(\varepsilon_t\) are unit normal variables and are mutually independent. The coefficient \(\alpha\) is an unknown coefficient, which represents the correlation between \(Y_t\) and credit cycle, \(L_t\).

We find the coefficient of each row of matrix (2) to minimize the weight, mean-squared discrepancies between the observed and fitted transition probabilities. We define

\[
P_t(i, j) = \Phi(y_{i,j+1}) - \Phi(y_{i,j})
\]

(4)

\[
P(y_{i,j+1}, y_{i,j}|L_t) = \Phi \left( \frac{y_{i,j+1} - \hat{c}L_t}{\sqrt{1-\alpha^2}} \right) - \Phi \left( \frac{y_{i,j} - \hat{c}L_t}{\sqrt{1-\alpha^2}} \right)
\]

(5)

where \(\Phi(\cdot)\) represents the standard normal cumulative distribution function, and equations (4) and (5) represent the observed and fitted transition probabilities of state \(i\) transferred to \(j\) observed in time-\(t\), respectively. The least square problem takes the form as in Belkin, Suchower and Forest (1998).

\[
\min_{L_t} \sum_i \sum_j \frac{n_{i,j}}{P_t(i, j)} \left( P_t(i, j) - P(y_{i,j+1}, y_{i,j}|L_t) \right)^2
\]

(6)

where \(n_{i,j}\) is the number of borrowers from initial state \(i\) transferred to state \(j\). In addition, the weighting factor is

\[
P_t(y_{i,j+1}, y_{i,j}|L_t) = [1 - P_t(y_{i,j+1}, y_{i,j}|L_t)]
\]

Therefore, we can get the fitted transition probability via equation (6). Then, we construct the fitted transition matrix as

\[
M = \begin{bmatrix}
m_{11} & m_{12} & \cdots & m_{1C} & m_{1,C+1} \\
m_{21} & m_{22} & \cdots & m_{2C} & m_{2,C+1} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
m_{C1} & m_{C2} & \cdots & m_{CC} & m_{C,C+1} \\
0 & 0 & \cdots & 0 & 1
\end{bmatrix}
\]

(7)

3.3 Risk Premium and Default Probability

In addition, this paper uses the risk-neutral probability approach to assess the credit risk of bank loans. For higher credit ratings, the historical default probability is almost zero, but the observed loan rates almost always imply a non-zero default probability in the risk-neutral world. Moreover, risk-neutral framework could contain unexpected default probability by estimating risk premium. Ideally, the implication is that we should match loan rates and observed transition matrices to obtain the risk premium. Accordingly, we conclude that a risk-neutral probability approach can also be applied to bank loans.

For the pricing of defaultable borrower, we need to consider the corresponding stochastic process \(\tilde{x} = \{\tilde{x}_t, t = 0, 1, 2, \cdots\}\) of credit rating under the risk-neutral probability measure. For valuation purposes, the fitted transition matrix needs to be transformed into a risk-neutral fitted transition matrix under an equivalent martingale measure where we let \(\tilde{M}\) denote such a matrix. The transition matrix under the new measure need not be Markovian if it is an absorbing Markov chain, which may not be time-homogeneous. Thus the fitted transition matrix under the risk-neutral probability measure is given by

\[
\tilde{M}(t, t+1) = \begin{bmatrix}
\tilde{m}_{11}(t,t+1) & \cdots & \tilde{m}_{1C}(t,t+1) & \tilde{m}_{1,C+1}(t,t+1) \\
\tilde{m}_{21}(t,t+1) & \cdots & \tilde{m}_{2C}(t,t+1) & \tilde{m}_{2,C+1}(t,t+1) \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{m}_{C1}(t,t+1) & \cdots & \tilde{m}_{CC}(t,t+1) & \tilde{m}_{C,C+1}(t,t+1) \\
0 & 0 & \cdots & 1
\end{bmatrix}
\]

(8)
satisfied here, together with the equivalence condition that \( \hat{m}_{ij}(t, t+1) > 0 \) if and only if \( m_{ij} > 0 \).

Note that the risk premium plays a crucial role for assessing the credit risk. The zero risk rate (risk-free rate) and risky rate (loan’s rate) can capture the credit risk of the bank loans for every rating class with the risk-neutral probability measure. First, let \( V_0(t, T) \) be the time-t price of a risk-free bond maturing at time \( T \), and \( V_j(t, T) \) be its higher risk, that is, risky counterpart for the rating class, \( i \). However, a loan does not lose all interest and principal when the borrower defaults. Realistically, we assume that a bank will receive some partial repayment even if the borrower goes into bankruptcy. Let \( \delta \) be the proportions of the loan’s principal and interest, which is collectable on default, where in general, \( \delta \) will be referred to as the recovery rate. If there is no collateral or asset backing, then \( \delta = 0 \). On the contrary, the recovery rate is \( 0 < \delta \leq 1 \).

As shown by Jarrow, Lando and Turnbull (1997), it can be assumed that \( \hat{m}_{ij}(t, t+1) = \hat{\lambda}_j(t) \cdot \hat{m}_{ij} \), \( i, j \in S \), and \( \hat{\lambda}_j(t) = \hat{\lambda}_j(t) \), for \( j \neq i \) and their procedure for risk premium as

\[
\hat{\lambda}_j(0) = \frac{V_0(0,1) - V_j(0,1)}{(1 - \delta)V_0(0,1)m_{i,C+1}}
\]

In equation (9), it is apparent that a zero or near-zero default probability, i.e., \( m_{i,C+1} \approx 0 \), would cause the risk premium estimate to explode, and it is also implied that the credit rating process (including default state) of every borrower is independent, which is inappropriate and irrational for bank loans. However, if the borrower defaults, then we should never estimate the default probability in the future. As a result, we modify the assumption that every borrower’s credit rating class is independent only before entering the default state. We redefine the risk premium as

\[
\hat{\ell}_i(t) = \frac{1}{1 - m_{i,C+1}} \sum_{j=1}^{C} \hat{m}_{ij}(0, t) \frac{V_j(0,1) - \delta V_j(0,1)}{(1 - \delta)V_0(0,1)}
\]

for \( i=1,2,\ldots,C \) and \( t=1,\ldots,T \)

\[
\hat{\Lambda}(0, t+1) = \hat{\Lambda}(0, t) \hat{\Lambda}(t, t+1)
\]

where \( \hat{m}_{ij}(0, t) \) are the components of the inverse matrix \( \hat{\Lambda}^{-1}(0, t) \) and \( \hat{\Lambda}(0, t) \) will be invertible. Note that \( m_{i,C+1} \) is the transition probability of fitted transition matrix by procedure as above. The denominator of equation (10) is not \( m_{i,C+1} \), but \( (1 - m_{i,C+1}) \), which can avoid the problem in equation (9). For equation (11), \( \hat{\Lambda}(t, t+1) = \Lambda(t) \cdot \hat{\Lambda}(t) \) and \( \Lambda(t) \) is the \((C \times C)\) diagonal matrix with diagonal components being the risk premium adjusted to \( \hat{\ell}_j(t) \), \( j \in S \). In particular, the risk premium of \( t = 0 \) is

\[
\hat{\ell}_i(0) = \frac{1}{1 - m_{i,C+1}} \times \frac{V_0(0,1) - \delta V_i(0,1)}{(1 - \delta)V_0(0,1)}
\]

for \( i=1,2,\ldots,C \)

Therefore, we can estimate the risk premium by a recursive method for all loan periods, \( t = 0, 1, \ldots, T \). On the whole, we also find that risk-neutral transition matrix varies over time to accompany the changes in the risk premium by equations (10) and (12). Then, we assume the indicator function to be

\[
l_{[i]} = \left\{ \begin{array}{ll}
1, & \text{if } 1 \in \{ \tau > T \} \text{ (not default before time } T \} \ (13)
\end{array} \right.
\]

Since the Markov processes and the interest rate are independent under the equivalent martingale measure, the value of the loan is equal to

\[
V_j(t, T) = V_0(t, T) \left\{ \hat{E}_1 \left[ I_{\{\tau > T\}} \right] + \hat{E}_\delta \left[ \delta I_{\{\tau > T\}} \right] \right\}
\]

\[
= V_0(t, T) \left\{ \hat{Q}_j(\tau > T) + \delta \left[ 1 - \hat{Q}_j(\tau > T) \right] \right\}
\]

where \( \hat{Q}_j(\tau > T) \) is the probability under the risk-neutral probability measure that the loan with rating \( i \) will not be in default before time \( T \). It is clear that

\[
\hat{Q}_j(\tau > T) = \frac{V_j(t, T) - \delta V_j(t, T)}{(1 - \delta)V_0(t, T)}
\]

\[
= \sum_{i=1}^{C} \hat{m}_{ij}(t, T) = 1 - m_{i,C+1}(t, T)
\]

which holds for time \( t \leq T \), including the current time, \( t = 0 \). Similarly, the default probability occurs before time \( T \) as

\[
\hat{Q}_j^*(\tau \leq T) = \frac{V_0(t, T) - V_j(t, T)}{(1 - \delta)V_0(t, T)}
\]

for \( i=1,\ldots,C \) and \( T=1,2,\ldots \)

There are four steps to assess credit risk of bank loans. First, we construct observed transition matrices based on the reports of rating agencies. Second, we get fitted transition probabilities by equation (5) to construct fitted transition matrix. Third, we extract risk premium via equation (10) and (12). Finally, we construct the transition matrix, matrix (8), condition on credit cycle and time-varying risk premium. Consequently, we estimate the default probability of bank loans by conditional Markov chain model that incorporates credit cycle and time-varying risk premium.

However, there are some limits to apply the proposed model. First, the model limit to application of borrowers with credit ratings. If a borrower without
credit rating, then the proposed model can’t work. Second, if the credit ratings mass in a rating class overly, the model may be failure. To exclude these limits, we expect that the proposed model could obtain more reliable estimate results for credit risk management.

4. Data

In Taiwan, there are two rating agencies, the Taiwan Rating and the Taiwan Economic Journal. The sample data come from two databases of the Taiwan Economic Journal (TEJ), including the Taiwan Corporate Risk Index (TCRI) and long and short-term bank loans.

The risk horizon is usually set at one year (Carty and Lieberman, 1996; Wei, 2003; Lu and Kuo, 2006). But this horizon is arbitrary and it mostly dictated by the availability of the data and financial reports processed by rating agencies. Therefore, we choose yearly data in the paper. The sample period is between 1997 and 2003.

The TCRI is a complete history of short and long-term rating assignments for Taiwan’s corporations. The definitions of the rating categories of TCRI for long-term credit are similar to Standard & Poor’s and Moody. TEJ applies a numerical class from 1 to 9 and D for each rating classification. The categories are defined in terms of default risk and the likelihood of payment for each individual borrower. Obligation rated number 1 is generally considered as being the lowest in terms of default risk, which is similar to the investment grade for Standard & Poor’s and Moody. Obligation rated number 9 is the most risky and the rating class D denotes the default borrower. Therefore, the rating categories used by TEJ, Standard & Poor’s and Moody are quite similar, though differences in opinions can lead in some cases to different ratings for specific debt obligations. On the other hand, since the borrowers’ obligation rated numbers are not consistent every year, we combine the number 1~4 as a new rating class, denoted as 1*. Similarly, we combine numbers 5~6 and 7~9 as two new rating classes, denoted as 2* and 3*, respectively. Thus, there are four rating classes, 1*, 2*, 3* and D.

The long and short-term bank loan database records all debts of corporations in Taiwan, including lender names, borrower names, rate of debt, and debt issuance dates. We can analyze the credit rating class of borrowers to investigate the credit risk of bank loans.

On the other hand, the risk-free rate is published by the Central Bank in Taiwan. We take the government bond’s yield as a proxy for the risk-free rate. The yields of government bonds for various maturities are published by the Central Bank in Taiwan. Since the maturity of bank loans and government bonds are different, we have to adjust the yields of government bonds, so we interpolate the yield of government bond whose maturity is the closest and take as the risk-free rate.

Finally, the recovery rate plays an important role for making lending decisions that serves as security for bank loans. In general, banks will set a recovery rate according to kinds, liquidity, and value of collateral before lending. Fons (1987) assumed a constant recovery rate of 0.41 according to the historic level. Longstaff and Schwartz (1995), and Briys and de Varenne (1997) also assumed a constant recovery rate. Carty and Lieberman (1996) assessed the recovery rate on a small sample of defaulted bank loans and found that it averaged over 71%. Copeland and Jones (2001) assumed that the recovery rate was equal to zero in all sample years. On the other hand, Lu and Kuo (2005, 2006) suggested taking the recovery rate as the exogenous variable from 0.1 to 0.9.

In particular, Altman, Resti and Sironi (2004) present a detailed review of default probability, recovery rate and their relationship. They find that most credit risk models treat recovery rate as an exogenous variable either structural-form models or reduced-form models. For structure-form models, recovery rate is exogenous and independent from the firm’s asset value (Kim, Ramaswamy and Sundaresan 1993; Hull and White, 1995; Longstaff and Schwartz, 1995). On the other hand, reduced-form models also assume an exogenous recovery rate that is either a constant or a stochastic variable independent from default probability (Litterman and Iben, 1991; Madan and Unal, 1995; Jarrow and Turnbull, 1995; Jarrow, Lando and Turnbull, 1997; Lando, 1998; Duffee, 1999). According to previous studies, there is no clear definition of the recovery rate, and the data on recoveries on defaulted loans is clearly incomplete. Consequently, we assume the recovery rate as exogenous variables from 0.1 to 0.9 in this paper.

In conclusion, we analyze the default risk for at least a one-year horizon and therefore exclude observations for short-term loans and incomplete data. We also exclude loans that have an overly low rate because they are likely to have resulted from aggressive accounting policies and bias the estimated results. Since the data without posting collateral are insufficient for gauging credit risk, we do not consider these loans.
That is, we analyze the credit risk of mid-and long-term loans with posting collateral for thirty-one domestic banks in Taiwan.

5. Empirical Results

In this paper, we estimate the credit risk of thirty-one domestic banks in Taiwan. We compare differences in credit risk between observed and fitted transition matrices.

Table 1 is summary statistic of our data. Visual inspections of Table 1, average loan rates and their corresponding government bond yields are 6.721% and 5.410%, respectively. In general, the risky rates are higher than risk-free rates. Furthermore, loan rates have greater volatility than that of risk-free rates. The average lending period is 6.488 years. Finally, loan rates and risk-free rates are not normality from the skewness and kurtosis. Figure 1 shows lending percentile for every industry in Taiwan. Averagely, the highest lending percentile is textile, which is the traditional industry in Taiwan.

Table 1. Summary Statistics

<table>
<thead>
<tr>
<th>Loan rate</th>
<th>Corresponding government bond</th>
<th>Lending period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>12.085%</td>
<td>7.907%</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.939%</td>
<td>1.206%</td>
</tr>
<tr>
<td>Mean</td>
<td>6.721%</td>
<td>5.410%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.820%</td>
<td>1.313%</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.960</td>
<td>1.394</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.516</td>
<td>-0.356</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.353</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Observed Transition Matrix, 1998-2003

<table>
<thead>
<tr>
<th>Rating at the end of year</th>
<th>Initial Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1*</td>
</tr>
<tr>
<td>1*</td>
<td>0.707</td>
</tr>
<tr>
<td>2*</td>
<td>0.035</td>
</tr>
<tr>
<td>3*</td>
<td>0.004</td>
</tr>
</tbody>
</table>
Table 3. Fitted Transition Matrix, 1998-2003

<table>
<thead>
<tr>
<th>Initial Rating</th>
<th>Rating at the end of year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1*</td>
</tr>
<tr>
<td>1*</td>
<td>0.781</td>
</tr>
<tr>
<td>2*</td>
<td>0.034</td>
</tr>
<tr>
<td>3*</td>
<td>0.016</td>
</tr>
<tr>
<td>( \hat{\alpha} )</td>
<td></td>
</tr>
</tbody>
</table>

Third, we estimate the time-varying risk premium to transform fitted transition matrix into risk-neutral fitted transition matrix via equations (10) and (12). Therefore, we list the average risk premium in Table 4. If the risk premium were to be plotted against the ratings, a skewed, U-shaped curve would emerge, with the trough corresponding to rating class 2*. We find that when the risk premium is exactly unity, the default probability will remain unchanged when the risk-neutral probability measurement is performed, i.e., \( m_{ij} = \hat{m}_{ij}, \forall i \). A risk premium smaller than 1.0 means \( m_{ij} < \hat{m}_{ij}, \forall i. \) From equation (10) and (12), the smaller risk premium estimate compensates for the bigger discrepancy between default probabilities under the real and the risk-neutral world. Thus we extract risk premium and incorporate that into fitted transition matrix. The fitted transition matrix under risk-neutral probability measure is shown in Table 5.

Averagely, the credit rating 2* have smaller risk premium in Table 4. Ideally, we can conjecture that there is larger discrepancy between default probabilities in real and neutral world. From Table 3 and 5, default probabilities in credit ratings 2* have bigger differences.

Table 4. Average Risk Premium

<table>
<thead>
<tr>
<th>Rating (Years)</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>0.955</td>
<td>0.968</td>
<td>0.984</td>
<td>0.975</td>
<td>0.956</td>
<td>0.973</td>
<td>0.969</td>
</tr>
<tr>
<td>2*</td>
<td>0.951</td>
<td>0.870</td>
<td>0.723</td>
<td>0.780</td>
<td>0.793</td>
<td>0.885</td>
<td>0.834</td>
</tr>
<tr>
<td>3*</td>
<td>0.941</td>
<td>0.994</td>
<td>0.999</td>
<td>0.995</td>
<td>0.993</td>
<td>0.999</td>
<td>0.986</td>
</tr>
</tbody>
</table>

Table 5. Fitted Transition Matrix under Risk-neutral Probability Measure, 1998-2003

<table>
<thead>
<tr>
<th>Initial Rating</th>
<th>Rating at the end of year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1*</td>
</tr>
<tr>
<td>1*</td>
<td>0.752</td>
</tr>
<tr>
<td>2*</td>
<td>0.028</td>
</tr>
<tr>
<td>3*</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Finally, we assess the average default probability of thirty-one banks by equation (16). Table 6 and Figure 2 show default probabilities of observed and risk-neutral fitted transition matrices, respectively. The observed default probabilities are estimated via historical credit ratings. In 1998, the observed default probability is higher resulted from Asian crisis and many firms are downgraded. However, if we consider credit cycle and risk premium, we find that default probabilities in 2000 and 2001 are higher than default probabilities in other years.

Although Lu and Kuo (2006) have been estimate credit risk of bank loans in Taiwan, they did not consider credit cycle. They adopt unconditional Markov chain model. Comparing with results of Lu and Kuo (2006), our results are somewhat differences. Our empirical results are all higher than Lu and Kuo (2006) and we conjecture that differences may be resulted from credit cycle.

In addition, we include the non-performing loan (NPL) ratio in Table 6. The NPL refers to loan accounts the principle and interest of which have become past due or those which exceeded the due date. The NPL ratio is equal to NPL divided by the total loan. Comparing with the NPL ratio, estimated results and the NPL ratio are all highest in 2001. Since the NPL ratio and default probability are concepts of ex-ante and ex-post, respectively. As a result, the NPL ratios are always lower than estimated results. In spite of the estimated results are not the same as the NPL ratios, we find that risk-neutral fitted default probabilities have more power to assess credit risk. For example, observed default probability in 2002 is 3.7% lower than the NPL ratio that is unreasonable and may be underestimated. Then, we compare empirical results and business cycle in Taiwan that is explained below.

One contribution of this paper is that it incorporates the credit cycle and time-varying risk premium into the transition matrix, which has never been discussed in previous studies. The credit cycle will be positive in good years, and negative in bad years. Figure 3 represents the historical movement of credit cycle that describes past credit conditions not evident in the observed transition matrices. On the other hand, Figure 4 shows the total scores of monitoring indicators of Taiwan from 1998 to 2003. The cyclical trough occurs in 2000-2001 that was the period in economic recession of Taiwan.

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10 This results is similar to Kijima and Komoribayashi (1998) and Wei (2003), who using a different set of data.

11 The total scores of monitoring indicators of Taiwan is composed of monetary aggregatesMIb, direct and indirect finance, bank clearing, remittance, stock price index, manufacturing new order index, exports, industrial production index, manufacturing inventory to sale ratio, nonagricultural employment, export price index, and manufacturing output price index.
Table 6. Default Probability

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed default probability</td>
<td>0.151</td>
<td>0.066</td>
<td>0.079</td>
<td>0.080</td>
<td>0.037</td>
<td>0.040</td>
</tr>
<tr>
<td>Risk-neutral fitted default probability</td>
<td>0.134</td>
<td>0.137</td>
<td>0.167</td>
<td>0.171</td>
<td>0.160</td>
<td>0.136</td>
</tr>
<tr>
<td>NPL</td>
<td>0.0437</td>
<td>0.0488</td>
<td>0.0534</td>
<td>0.0748</td>
<td>0.0612</td>
<td>0.0433</td>
</tr>
</tbody>
</table>

Note: NPL is non-performing loan ratio of domestic banks in Taiwan. The NPL data is obtained from Bureau of Monetary Affairs, Financial Supervisory Commission, Executive Yuan, R. O. C.

From Figures 3 and 4, we find that the credit cycle drops below zero while Taiwan’s business cycle from peak to trough in 2000-2001. From Table 6 and Figure 2, we find that default probabilities in 2000-2001 are higher than other years. On the whole, the relative high proportion of borrowers together with 2000-2001 credit slump accounts for a high number of defaults that may be due to the business cycle. That is, the credit risk in 2000-2001 is higher than other years. On the other hand, credit cycle has stayed positive and credit conditions have remained benign, and the default probabilities are low during other periods. Consequently, the model for estimating the default probability is accurate and reliable. Peak to trough in 2000-2001. From Table 6 and Figure 2, we find that default probabilities in 2000-2001 are higher than other years. On the whole, the relative high proportion of borrowers together with 2000-2001 credit slump accounts for a high number of defaults that may be due to the business cycle. That is, the credit risk in 2000-2001 is higher than other years. On the other hand, credit cycle has stayed positive and
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6. Conclusion

This paper focuses on providing a formal model to assess the credit risk of bank loans for thirty-one domestic banks in Taiwan over the period 1998 to 2003. We extend Markov chain model by incorporating credit cycle and risk premium, which is called conditional Markov chain model. The proposed model will get more reliable estimate results for risk management.

The main conclusions and contributes can be drawn in the following aspects. First, the model incorporates the credit cycle into the observed transition matrix, so-called fitted transition matrix. In addition, we compare the differences between the observed and fitted transition matrix. The empirical result shows that the observed transition matrix will underestimate the default probability of risky rating class may be resulted from ignoring credit cycle.

Second, we relax the assumption of risk premium as a time-varying parameter that is assumed as a constant parameter in Jarrow, Lando and Turnbull (1997) and Wei (2003). Comparing with business cycle, we find that the empirical result is more reliable after considering credit cycle and risk premium. On the other hand, we believe that the proposed model can capture the influence of business cycle. Finally, the procedure is easy to follow and implement. We recommend that the method can apply to assess the credit risk of other financial institutions, such as a bills finance company. On the whole, we expect that this paper can help banks estimate their credit risk more carefully and develop an effective tool for any credit review process of financial institutions.

For subsequent research, the availability of data on recovery rate over time can be the way to develop a dynamic measure of credit risk for financial institutions. On the other hand, further studies also can develop satisfactory way to derive the exposure and loss distribution when default occurs.

Acknowledgments

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Reference


